Problem Set 6

due: June 23

1. Recommended practice problems from Sections 4.7, 4.9, 5.1–5.3, Chapter 4 Review, and Appendix E.

2. Evaluate the following sums:

(a) $\displaystyle\sum_{k=1}^{m}\left(\sum_{l=1}^{n}\frac{k}{l(l+1)}\right)$

(a) $\displaystyle\sum_{k=1}^{n}\left(\frac{2017}{\sum_{l=1}^{k} l}\right).$

3. Find the limit:

$$\lim_{n\to\infty}\left(\sqrt{\left(\sum_{i=1}^{n} i\right)}-\frac{n\sqrt{2}}{2}\right).$$

4. Let f be a function defined (piecewise) on the interval $[1, 2017)$ by the formula

$$f(x) = \frac{2}{k(k+2)}, \quad \text{for } x \in [k, k+1), \text{ where } k = 1, \ldots, 2016.$$

Find the area of the region bounded by the graph of f, the x-axis and the lines $x = 1$ and $x = 2017$.

Bonus. (a) Let f be *any* linear function on a closed interval $[a, b]$ (i.e., a function of the form $f(x) = Ax + B$ for some $A, B \in \mathbb{R}$). Show that

$$\int_a^b f(x)\,\mathrm{d}x = (b-a)\cdot f\left(\frac{a+b}{2}\right).$$

(b) Let g be a function twice differentiable on a closed interval $[a, b]$. Prove that if g is concave up then

$$\int_a^b g(x)\,\mathrm{d}x \ \leq\ (b-a)\cdot\frac{g(a)+g(b)}{2}.$$