

Assignment 1
due: September 28

1. Find the domain and range, and sketch the graph of the function:

(a) $f(x) = 2 - 2^{-x}$

(b) $g(x) = e^{|x+1|}$.

2. Find the domain and range of the function f . Justify your answer.

(a) $f(x) = \ln(\arctan(e^x - 2018))$

(b) $f(x) = \sin(\sqrt{\pi x - 4x^2})$

(c) $f(x) = \ln\left(\frac{e^x + e^{-x}}{2}\right)$.

3. (a) Use the definition of injectivity to show that, if functions f and g are injective, then so is the composite function $g \circ f$.

(b) Use the definition of the inverse function to show that, if functions f and g are injective, then the inverse $(g \circ f)^{-1}$ is equal to the function $f^{-1} \circ g^{-1}$.

(c) Use part (b) to find the inverse of the function

$$h(x) = \frac{\ln x + 3}{5 - \ln x}.$$

4. Determine which of the following functions are injective. Justify your answers.

(a) $y = \frac{4x - 1}{2x + 3}$

(b) $y = \frac{1}{1 + e^{-x}}$

(c) $y = x^3 - x$

(d) $y = \arcsin(\ln x)$.

5. Find the inverses of functions from Problem 4, whenever possible. For each inverse function, determine its domain and range. Justify your answers.