

Assignment 2
due: October 15

1. Use the Squeeze Theorem to find the following limits. Justify your answers.

(a) $\lim_{x \rightarrow 1^-} \sqrt{1-x^2} \cdot \sin^5(\ln(1-x^2))$

(b) $\lim_{x \rightarrow \infty} \sin\left(\frac{\pi}{x}\right) \cdot e^{\cos(\pi x)}$

(c) $\lim_{x \rightarrow \infty} \sqrt{x} \cdot \sin\left(\frac{1}{x}\right)$.

2. Show that $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$.

[Hint: Use the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.]

3. Find $\lim_{x \rightarrow 0} \frac{\sin(\sin(\sin x))}{x}$. Justify your answer.

4. Use the Intermediate Value Theorem to show that the following equations have a solution in a given interval I . Justify your answers.

(a) $x^5 - 4x^2 + e^x = 0$, $I = (-1, 1)$;

(b) $e^{\sin x} - \sin(\cos x) = 1$, $I = (0, \frac{3\pi}{2})$.

[Hint: Consider the restriction to a suitable subinterval of I .]

5. Use the definition of derivative to find derivatives of the following functions:

(a) $f(x) = \cos x$

(b) $g(x) = \frac{1}{x^2}$.