Practice Term Test 2

1. True or False:

   (i) If $\mathbf{T}(t)$ is the unit tangent vector of a smooth curve, then the curvature is $\kappa = \|d\mathbf{T}/dt\|$.

   A: YES B: NO

   (ii) Suppose $f(x)$ is twice continuously differentiable. At an inflection point of the curve $y = f(x)$, the curvature is 0.

   A: YES B: NO

   (iii) If $\|\mathbf{r}(t)\| = 1$ for all t, then $\|\mathbf{r}'(t)\|$ is a constant.

   A: YES B: NO

   (iv) The osculating circle of a curve C at a point has the same unit tangent vector, normal vector, and curvature as C at that point.

   A: YES B: NO

   (v) If $f(x, y)$ is a function and (a, b) is in the domain of f, then $\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$.

   A: YES B: NO

   (vi) The linearization of $f(x, y) = e^x \cos(xy)$ at the point $(0, 0)$ is $L(x, y) = x + 1$.

   A: YES B: NO

   (vii) The linearization of $f(x, y) = \dfrac{y-1}{x+1}$ at the point $(0, 0)$ is $L(x, y) = x + y - 1$.

   A: YES B: NO

   (viii) If $f(x, y)$ has continuous second-order partial derivatives, then $\dfrac{\partial^2 f}{\partial x \partial y}(x, y) - \dfrac{\partial^2 f}{\partial y \partial x}(x, y) = 0$ for all (x, y).

   A: YES B: NO

   (ix) There exists a function $f(x, y)$ with continuous second-order partial derivatives, such that $\dfrac{\partial f}{\partial x}(x, y) = x + y^2$ and $\dfrac{\partial f}{\partial y}(x, y) = x - y^2$ for all $(x, y) \in \mathbb{R}^2$.

   A: YES B: NO

2. The minimum curvature of the curve $\mathbf{r}(t) = \dfrac{\cos(3t^2)}{4}\,\mathbf{j} - \dfrac{\sin(3t^2)}{4}\,\mathbf{k}$ is equal to

A: 12	B: 4	C: 3	D: 2	E: 0

3. If the position function of a particle is $\mathbf{r}(t) = t^2\mathbf{j} + \dfrac{1}{t^2}\mathbf{k}$, then its acceleration at time $t = 1$ is

A: $\langle 0, 1, 2\rangle$	B: $\langle 1, 2, 6\rangle$	C: $\langle 2, 0, 6\rangle$	D: $\langle 6, 2, 0\rangle$	E: $\langle 0, 2, 6\rangle$

4. At what point of the curve $\mathbf{r}(t) = t^3\mathbf{i} + 3t\mathbf{j} + t^4\mathbf{k}$ is the normal plane parallel to the plane $6x + 6y - 8z = 2023$?

A: $P(1, 3, 1)$	B: $P(-1, -3, 1)$	C: $P(-1, 3, 1)$	D: $P(3, 3, -4)$	E: $P(8, 6, 16)$

5. If $f(x, y) = \dfrac{5y^4 \cos^2(2x)}{x^4 + y^4}$, then $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$ is equal to

A: 5	B: $\frac{5}{2}$	C: $\frac{5}{4}$	D: 0	E: the limit does not exist

6. If $f(x, y) = y \sin(x - y)$, then $\displaystyle\lim_{(x,y)\to(\pi/2,\pi)} f(x, y)$ is equal to

A: $-\pi$	B: $-\pi/2$	C: 0	D: $\pi/2$	E: the limit does not exist

7. Find the unit tangent and unit normal vectors of the curve $\mathbf{r}(t) = \langle t, \frac{t^2}{2}, t^2\rangle$ at the point $t = 0$.

8. Prove that the osculating plane at every point on the curve $\mathbf{r}(t) = \langle t + 2, 1 - t, \frac{t^2}{2}\rangle$ is the same plane.

9. Suppose that a projectile is fired with an initial speed v_0 and angle of elevation α from the initial position at the origin, and its acceleration function is $\mathbf{a}(t) = -g\mathbf{j}$. Show that the projectile reaches three-quarters of its maximum height in half the time needed to reach its maximum height.

10. Given that $f(x, y) = 2y + e^{-x}$, $x(s, t) = s - \ln(\sin t)$, and $\dfrac{\partial^2 y}{\partial s^2}(0, \pi/2) = 1$, find $\dfrac{\partial^2 f}{\partial s^2}(0, \pi/2)$. Justify your answer.

11. Find the linearization $L(x, y)$ of the function $f(x, y) = y + \sin\left(\dfrac{x}{y}\right)$ at the point $(0, 3)$.