Adjoint functors between quasicategories

Luis Scoccola

Reading seminar on Higher Category Theory
University of Western Ontario, Fall 2016

The goal of this talk is to understand adjunctions in the higher-categorical setting. We do this by introducing two different definitions and then proving that they are equivalent. The references are:

- [RV15] for the 2-categorical point of view and many applications of the 2-categorical approach.
- [Lur09, Chapter 5] for a more comprehensive theory of adjunctions, (co)limits and (locally) presentable quasicategories.

1 2-categorical adjunctions

Adjunctions are inherently a 2-categorical concept. We will therefore define a suitable 2-category of quasicategories and use it to obtain the notion of adjunction.

Definition 1. We denote the sub-2-category of sSet^{τ_1} spanned by quasicategories by qCat^{τ_1}.

Since the nerve functor maps categories to quasicategories and it is fully faithful, its image in sSet^{τ_1} can be naturally identified with the 2-category of categories so the 2-category theory of quasicategories generalizes the 2-category theory of categories.

Recall that a 1-cell $u : A \to B$ in a 2-category is an equivalence if and only if precomposing with u induces an equivalence of categories $\hom(B, C) \to \hom(A, C)$ for every 0-cell C. This implies that a functor between quasicategories is a weak equivalence in the Joyal model structure if and only if it is an equivalence in the 2-category qCat^{τ_1}.

Definition 2. An adjunction in a 2-category consists of two objects C, D, two 1-cells $f : C \to D$, $g : D \to C$ and two 2-cells $\eta : \mathsf{Id}_C \Rightarrow g \circ f$, $\varepsilon : f \circ g \Rightarrow \mathsf{Id}_D$ that satisfy the triangle identities:

$$\begin{tikzcd}
& D \arrow[rr, equal] \arrow[dr, "g"] \arrow[d, phantom, "\Downarrow\eta"] & & D \\
C \arrow[ur, "f"] \arrow[rr, equal] & {} & C \arrow[ur, "f"'] \arrow[u, phantom, "\Downarrow\varepsilon"] &
\end{tikzcd}
\quad = \quad
\begin{tikzcd}
C \arrow[r, bend left, "f"] \arrow[r, bend right, "f"'] \arrow[r, phantom, "\mathsf{Id}_f"] & D
\end{tikzcd}$$

$$
\begin{tikzcd}
& C \arrow[rr, equal] \arrow[dr, "f"] \arrow[d, phantom, "\Uparrow\varepsilon"] & & C \\
D \arrow[ur, "g"] \arrow[rr, equal] & {} & D \arrow[ur, "g"'] \arrow[u, phantom, "\Uparrow\eta"] &
\end{tikzcd}
\quad = \quad
\begin{tikzcd}
C \\
D \arrow[u, bend left=40, "g"] \arrow[u, bend right=40, "g"'] \arrow[u, phantom, "\overset{\mathsf{Id}_g}{=\!=}"]
\end{tikzcd}
$$

Definition 3. An *adjunction between quasicategories* is an adjunction in the 2-category qCat^{τ_1}. We will sometimes refer to these adjunctions as 2*-categorical adjunctions*.

Using the fact that any pair of edges that get identified when passing to the homotopy category of a quasicategory are in fact connected by a 2-simplex in the quasicategory, we deduce that in qCat^{τ_1} for any two composable 2-cells a, b and their composition c in $\mathsf{qCat}^{\tau_1}(A, B)$ there is an actual representative $\alpha : \Delta^2 \times A \to B$ that witnesses the composition $a \circ b = c$.

Remark 4. Having an adjunction between quasicategories is the same as having the following data:

- Two quasicategories C, D.
- Two maps between the quasicategories $f : C \rightleftarrows D : g$.
- Two natural transformations $\eta : D \times \Delta^1 \to D$, $\varepsilon : C \times \Delta^1 \to C$, of the form $\mathsf{Id}_D \Rightarrow f \circ g$, and $g \circ f \Rightarrow \mathsf{Id}_C$ respectively.
- Two 2-simplices α, β, one in D^C and one in C^D that witness the triangular identities:

$$
\begin{tikzcd}
& f \circ g \circ f \arrow[dr, "f\varepsilon"] \arrow[d, phantom, "\alpha"] & \\
f \arrow[ur, "\eta f"] \arrow[rr, "\mathsf{Id}_f"'] & {} & f
\end{tikzcd}
\qquad
\begin{tikzcd}
& g \circ f \circ g \arrow[dr, "\varepsilon g"] \arrow[d, phantom, "\beta"] & \\
g \arrow[ur, "g\eta"] \arrow[rr, "\mathsf{Id}_g"'] & {} & g
\end{tikzcd}
$$

Example 5. Since the 2-category of categories is a full-2-subcategory of the 2-category of quasicategories the quasicategorical adjunctions between the nerve of two categories are exactly the 1-categorical adjunctions between the two categories. Moreover, we have canonical representatives for the unit and couint, applying the nerve functor to the 1-categorical unit and counit.

Example 6. A simplicially enriched adjunction between locally Kan simplicial categories (the fibrant objects in Bergner's model structure) gives rise to an adjunction of quasicategories by applying the homotopy coherent nerve. As in the previous example, this follows by functoriality. We also have canonical representatives for the unit and counit, applying the homotopy coherent nerve to the enriched unit and counit.

Example 7. A simplicially enriched Quillen adjunction between simplicial model categories induces an adjunction between the associated quasicategories. To do this we first have to restrict the adjunction to the fibrant-cofibrant objects and then apply the homotopy coherent nerve. This is not immediate, a complete argument is in [RV15, Theorem 6.2.1].

A nice consequence of the definition is that all the 2-categorical facts and arguments are applicable. For example the following two facts follow from basic 2-category theory.

Proposition 8. *Given two adjunctions $f : C \rightleftarrows D : g$ and $f' : D \rightleftarrows E : g'$ we can compose them to get and adjunction $f'f : C \rightleftarrows E : gg'$.* ∎

Proposition 9. *Any equivalence w in a 2-category can be promoted to an adjoint equivalence in which the equivalence w can be taken to be either the left or the right adjoint.* ∎

Recall that exponentiation by a fixed simplicial set X induces a simplicially enriched functor $(-)^X : \mathsf{sSet} \to \mathsf{sSet}$ and that moreover it restricts to a simplicially enriched functor $(-)^X : \mathsf{qCat} \to \mathsf{qCat}$ since exponentials of quasicategories are quasicategories. This functor induces a 2-functor $(-)^X : \mathsf{qCat}^{\tau_1} \to \mathsf{qCat}^{\tau_1}$. Using the fact that 2-functors preserve adjunctions in 2 categories we deduce the following result.

Proposition 10. *Given $f : C \rightleftarrows D : g$ an adjunction between quasicategories, a simplicial set X and a quasicategory E we have two induced adjunctions between quasicategories:*

$$f^X : C^X \rightleftarrows D^X : g^X$$
$$E^g : E^C \rightleftarrows E^D : E^f$$

∎

2 Cartesian fibrations

For our second definition of an adjunction we need to introduce the notion of a Cartesian fibration. To this end we first need to say what a Cartesian edge in a quasicategory is.

Definition 11. Given an inner fibration $p : X \to S$ between simplicial sets, we say that an edge $f : x \to y$ in X is *p-Cartesian* if the induced map to the pullback:

$$\begin{tikzcd}
X/f \arrow[dr, dashed] \arrow[drr, bend left] \arrow[ddr, bend right, "p"'] & & \\
& X/y \times_{S/p(y)} S/p(f) \arrow[r] \arrow[d] & X/y \arrow[d, "p"] \\
& S/p(f) \arrow[r] & S/p(y)
\end{tikzcd}$$

is a trivial Kan fibration.

Dually, an edge is p-coCartesian if it is p^{op}-Cartesian, where $p^{op} : X^{op} \to S^{op}$.

Remark 12. To understand the previous definition let us see how the vertices of the simplicial sets involved look like.

A vertex in X/f is a (filled) 2-simplex (in X) of the form:

$$\begin{tikzcd} x' \arrow[d, "n"'] \arrow[dr, "m"] & \\ x \arrow[r, "f"'] & y \end{tikzcd}$$

while a vertex of $X/f \times_{S/p(y)} S/p(f)$ consists of a horn in X/f sitting on top of a (filled) 2-simplex in S:

$$\begin{tikzcd} x' \arrow[dr, "m"] & \\ x \arrow[r, "f"'] & y \end{tikzcd}$$

$$\begin{tikzcd} p(x') \arrow[d, "n'"'] \arrow[dr, "p(m)"] & \\ p(x) \arrow[r, "p(f)"'] & p(y) \end{tikzcd}$$

So intuitively the equivalence in the definition of Cartesian morphism is telling us that f is Cartesian if every time we can fill a horn (with f in the place $[1,2]$) once we map it to S then we can lift the filling *in an essentially unique way*.

Next, we will give two ways to characterize Cartesian edges. The second one is particularly conceptual and makes the analogy with the 1-categorical definition very clear.

Consider the map $\varphi : X/f \to X/f \times_{S/p(y)} S/p(f)$ in the definition of p-Cartesian edge. The map lives over X, by projecting the source of the 1-simplices in X/f:

$$\begin{tikzcd} X/f \arrow[rr, "\varphi"] \arrow[dr, "(-)|\Delta^{\{0\}}"'] & & X/f \times_{S/p(y)} S/p(f) \arrow[dl, "(-)|\Delta^{\{0\}}"] \\ & X & \end{tikzcd}$$

Restricting to a vertex $x \in X$, we obtain the induced map on the fibers $\varphi_x : \{x\} \times_C C/f \to \{x\} \times_C (C/z \times_{D/p(z)} D/p(f))$.

Proposition 13. *Let $p : C \to D$ be an inner fibration between quasicategories and $f : y \to z$ a morphism in C. Then the following are equivalent:*

1. *The morphism f is p-Cartesian*

2. *For every x vertex of C, the induced map $\varphi_x : \{x\} \times_C C/f \to \{x\} \times_C (C/z \times_{D/p(z)} D/p(f))$ is a trivial fibration.*

We will use the following theorem from Dinesh's talk.

Theorem 14. *If $f : X \to Y$ is an inner fibration of simplicial sets, $S \hookrightarrow T$ is an inclusion and $t : T \to X$ is simplicial map, then the projection:*

$$p : X/T \to X/S \times_{Y/S} Y/T$$

is a right fibration. ■

Proof of Proposition 13. By definition, f is p-Cartesian if and only if $\varphi : C/f \to C/z \times_{D/p(z)} D/p(f)$ is a trivial fibration. On one hand, by Theorem 14, φ is a right fibration and thus it is trivial if and only if the fibers are contractible Kan complexes. On the other hand, φ_x is a right fibration since it is a pullback of φ, so it is a trivial fibration if and only if it has contractible fibers [Lur09, Lemma 2.1.3.4]. Now notice that any fiber of φ is isomorphic to a fiber of φ_x for some x, and conversely that for any x any fiber of φ_x is isomorphic to a fiber of φ. This means that φ has contractible fibers if and only if φ_x has contractible fibers for every x. ■

Using the above result we get the following characterization.

Proposition 15. *Given an inner fibration $p : C \to D$ between quasicategories and an edge $f : y \to z$ in C, f is p-Cartesian if and only if the following diagram is a homotopy pullback for every $x \in C$:*

$$\begin{tikzcd}
\{x\} \times_C C/f \arrow[r] \arrow[d] & \{x\} \times_C C/z \arrow[d] \\
\{p(x)\} \times_D D/p(f) \arrow[r] & \{p(x)\} \times_D D/p(z)
\end{tikzcd}$$

Proof. Consider the pullback square:

$$\begin{tikzcd}
\{x\} \times_C C/z \times_D D/p(f) \arrow[r] \arrow[d] & \{x\} \times_C C/z \arrow[d] \\
\{p(x)\} \times_D D/p(f) \arrow[r] & \{p(x)\} \times_D D/p(z)
\end{tikzcd}$$

The vertical map on the right is a Kan fibration since it is a pullback of the map $X/z \to S/p(z) \times_S X$ which is a right fibration (by Theorem 14) with codomain a Kan complex (as a right mapping space). This means that the square is in fact a homotopy pullback.

The square in the statement is induced by precomposing with $\varphi_x : \{x\} \times_C C/f \to \{x\} \times_C C/x \times_D S/p(f)$. So the square in the statement is a homotopy pullback if and only if φ_x is a trivial fibration. ■

Recall that given two vertices x, y in quasicategory S the right mapping space $\hom^R_S(x, y)$ is the simplicial set:

$$\hom^R_S(x,y)_n = \left\{ \Delta^{n+1} \xrightarrow{f} S \;\middle|\; f|_{\Delta^{\{0\dots n\}}} = x, f\,|_{\Delta^{\{n+1\}}} = y \right\}.$$

Notice that this simplicial set can also be described as the pullback $\{x\} \times_S S/y$. Using this and the identification $\{x\} \times_C C/f \simeq \{x\} \times_C C/y$ we get.

Corollary 16. *Under the hypothesis of Proposition 15 f is a Cartesian edge if and only if the following is a homotopy pullback square:*

$$\begin{tikzcd} \hom_C(x,y) \arrow[r, "f_*"] \arrow[d, "p"'] & \hom_C(x,z) \arrow[d, "p"] \\ \hom_D(p(x),p(y)) \arrow[r, "p(f)_*"] & \hom_D(p(x),p(z)) \end{tikzcd}$$

We now extend the concept of Cartesian functor to the higher categorical setting.

Definition 17. A map $p : X \to S$ of simplicial sets is a *Cartesian fibration* if it satisfies the following:

- The map p is an inner fibration.
- For every edge $g : s \to t$ of S and every lift $\tilde{t}$ of t there exists a p-Cartesian lift $\tilde{g}$ of g.

Dually, a map $p : X \to S$ is a *coCartesian fibration* if $p^{op} : X^{op} \to S^{op}$ is a Cartesian fibration.

Notice that a functor between 1-categories is a Grothendieck fibration if and only if its nerve is a Cartesian fibration.

3 Adjunctions via Cartesian fibrations

To motivate the next definition let us recall for a moment the Grothendieck construction in the particular case when the domain category is the one-arrow category. The Grothendieck construction of a 1-functor $\mathcal{F} : \Delta^1 \to \mathsf{Cat}$ is a Cartesian functor $\mathcal{P} : M \to \Delta^1$. The functor $\mathcal{F}$ is nothing but two categories $\mathcal{C}, \mathcal{D}$ together with a functor $\mathcal{G} : \mathcal{C} \to \mathcal{D}$ between them. The following definition describes the relation between $\mathcal{G}$ and p.

Definition 18. Given a Cartesian fibration $p : M \to \Delta^1$ and two equivalences of quasicategories $h_0 : C \to p^{-1}\{0\}, h_1 : C \to p^{-1}\{1\}$ a simplicial map $g : D \to C$ is *associated* to M if we have a commutative diagram:

$$\begin{tikzcd} D \times \Delta^1 \arrow[r, "s"] \arrow[dr] & M \arrow[d] \\ & \Delta^1 \end{tikzcd}$$

such that:

- $s|D \times \{1\} = h_1$.
- $s|D \times \{0\} = h_0 \circ g$.

- $s|\{x\} \times \Delta^1$ is a p-Cartesian edge of M for every x vertex of D.

Dually a map f is *coassociated* to a map p if f^{op} is associated to p^{op}.

The following is a consequence of the straightening $\dashv$ unstraightening construction.

Proposition 19 ([Lur09, 5.2.1.6]). *Given two quasicategories C, D, there is a bijective correspondence between equivalence classes of simplicial maps $D \to C$ and equivalence classes of Cartesian fibrations $p : M \to \Delta^1$ equipped with equivalences $C \to M_0 := p^{-1}\{0\}, D \to M_1 := p^{-1}\{1\}$.*

Moreover, $g : D \to C$ is associated to $p : M \to \Delta^1$ if and only if they correspond to each other in the bijection. ■

We are ready to define adjunctions between quasicategories in the sense of Lurie.

Definition 20. Given two quasicategories C and D, an *adjunction* between C and D is a map $q : M \to \Delta^1$ which is both a Cartesian fibration and coCartesian fibration, together with equivalences $C \to M_0$ and $D \to M_1$.

In this case, let $f : C \to D$ and $g : D \to C$ be functors coassociated and associated to M respectivelly, then we say f is a *left adjoint* and g is a *right adjoint*.

4 Equivalence of the definitions

To prove the equivalence between the two definitions we…introduce a third definition.

Definition 21. Given simplicial maps $f : C \to D$ and $g : D \to C$ between quasicategories, a *unit transformation* for (f, g) is a natural transformation $u : \mathsf{Id}_C \Rightarrow g \circ f$ (i.e. $u : C * \Delta^1 \to C$ such that $u|C * \Delta^{\{0\}} = \mathsf{Id}_c, u|C * \Delta^{\{1\}} = g \circ f$) such that for every pair of vertices $c \in C, d \in D$ the composition:

$$\hom_D(f(c), d) \xrightarrow{g} \hom_C(g(f(c)), g(d)) \xrightarrow{u_c^*} \hom_C(c, g(d))$$

is an isomorphism in $\mathsf{Ho}(\mathsf{sSet}_Q)$.

To prove that unit transformations are equivalent to adjunctions defined by correspondences we need the following proposition.

Proposition 22 ([Lur09, 3.1.2.1]). *Let $p : X \to S$ be a Cartesian fibration between simplicial sets and let K be any simplicial set. Then the induced map $p^K : X^K \to S^K$ is a Cartesian morphism and moreover: An edge in X^K is p^K-Cartesian if and only if it is point-wise p-Cartesian. This means that it is p-Cartesian when evaluated at each vertex of K.* ■

For simplicity in the proof of the following proposition we will assume that C and D are actually subcategories of M, i.e. $C = M_0$ and $D = M_1$.

Theorem 23 (Lurie). *For a pair of functors $f : C \rightleftarrows D : g$ between quasicategories the following are equivalent:*

1. *The functor f is left adjoint to g in the sense of Lurie.*

2. *There exists a unit transformation $u : \mathsf{Id}_C \Rightarrow g \circ f$.*

3. *f is left adjoint to g in the 2-categorical sense.*

Proof. We first show $(1) \Rightarrow (2)$. Suppose that (1), so we have an adjunction $p : M \to \Delta^1$ coassociated and associated to f and g respectively. Since M is coassociated to f we have a map:

$$\begin{aligned} F : C \times \Delta^1 &\to M \\ (c,0) &\mapsto c \\ (c,1) &\mapsto f(c) \end{aligned}$$

such that F_c is p-coCartesian for every c. And since it is also associated to g we have a map:

$$\begin{aligned} G : D \times \Delta^1 &\to M \\ (d,0) &\mapsto g(d) \\ (d,1) &\mapsto d \end{aligned}$$

such that G_d is p-Cartesian for every d. We want to construct a unit transformation for (f, g). Now we can consider the diagram:

$$\begin{tikzcd} C \times \Lambda^2_2 \arrow[r, "F'"] \arrow[d, hook] & M \arrow[d] \\ C \times \Delta^2 \arrow[r] & \Delta^1 \end{tikzcd}$$

where $F'|C \times \Delta^{[0,2]} = F$ and $F'|C \times \Delta^{[1,2]} = G \circ (f \times \mathsf{Id}_{\Delta^1})$, and the bottom map sends:

$$\begin{aligned} [0,2] &\mapsto [0,1] \\ [1,2] &\mapsto [0,1] \\ [0,1] &\mapsto [0] \end{aligned}$$

Since $F'|\{c\} \times \Delta[1,2]$ is p-Cartesian for every c we can solve the lifting problem with a map $F'' : C \times \Delta^2 \to M$ by Proposition 22. So we define a natural transformation $u : \mathsf{Id}_C \Rightarrow gf$ as the restriction $u := F''|C \times \Delta^{[0,1]}$. We must show that it is in fact a unit transformation.

Consider the following diagram:

$$\begin{tikzcd} \hom_D(f(c), d) \arrow[r, "g"] \arrow[d, "F(c)^*"'] & \hom_C(g(f(c)), g(d)) \arrow[r, "u_c^*"] & \hom_C(c, g(d)) \arrow[d, "G(d)_*"] \\ \hom_M(c, d) \arrow[rr, equal] & & \hom_M(c, d) \end{tikzcd} \qquad (*)$$

If we want to be precise we can say that the objects in this diagram are right mapping spaces.$C \simeq M_0$ and $D \simeq M_1$. The diagram $(*)$ is homotopy commutative as the following diagram shows:

$$
\begin{tikzcd}
c \arrow[d, "u_c"'] \arrow[dr, "F(c)"] & \\
gf(c) \arrow[r, "G(f(c))"'] \arrow[d, "g(e)"'] \arrow[ur, phantom, "F''(c)"] \arrow[dr, phantom, "G(e)"] & f(c) \arrow[d, "e"] \\
g(d) \arrow[r, "G(d)"'] & d
\end{tikzcd}
$$

Commutativity is given by the fact that G is a natural transformation $g \Rightarrow \mathsf{Id}$ and u is a natural transformation $\mathsf{Id} \Rightarrow g \circ f$. Now, the simplicial set Δ^1 is contractible and thus, by Corollary 16 and its dual, both vertical maps are equivalences, since the pullback along an equivalence is an equivalence, and hence u is an equivalence.

For the $(2) \Rightarrow (1)$ use Proposition 19 to get a Cartesian fibration $p : M \to \Delta^1$ associated to g via a map $G : D \times \Delta^1 \to M$. Define $F : C \times \Delta^1 \to M$ to be a composition of natural transformations $G \circ (f \times \mathsf{Id}_{\Delta^1}) : C \times \Delta^1 \to M$ and $u : C \times \Delta^1 \to M$ (here we say *a* composition since this a composition of edges in the quasicategory M^C). We have to check two conditions:

1. $p : M \to \Delta^1$ is a coCartesian fibration.
2. f is co-associated to M via F.

We *claim* that 1. and 2. follow from $F(c)$ being p-coCartesian for every c, but let us postpone the proof of this claim for a moment. Consider again the diagram $(*)$, which is homotopy commutative. Since u is a unit transformation the top composition is an equivalence, and since g is Cartesian so is the right vertical map. This implies that the left vertical map is also an equivalence. Now the dual of Corollary 16 implies that $F(c)$ is p-coCartesian for every c, as we wanted to show.

To prove the *claim* notice that p is already an inner fibration. Consider then an edge in Δ^1 with a lift of its source c and choose $F(c)$ for the lifting of the edge, if $F(c)$ is p-coCartesian then the first condition follows. For the second condition we already have the commutative diagram of the definition of associated functor, so the condition again follows from $F(c)$ being p-coCartesian for every c.

Let us now prove $(3) \Rightarrow (2)$. Suppose we have a 2-categorical adjunction between quasicategories with unit and counit η and ε respectively. We want to prove that η is a unit transformation. To prove this consider the composites $\varphi = \eta_c^* \circ g : \hom_D(fc, d) \to \hom_C(c, gd)$ and $\theta = \varepsilon_{d*} \circ f : \hom_C(c, gd) \to \hom_D(fc, d)$. We have to show that φ is an equivalence, we will prove that θ is its homotopy inverse. For this consider $m \in \hom_C(c, gd)$ and the diagram:

$$\begin{tikzcd} c \arrow[r, "\eta_c"] \arrow[d, "m"'] & gfc \arrow[d, "gfm"] \\ gd \arrow[r, "\eta_{gd}"] \arrow[dr, equal] & gfgd \arrow[d, "g\varepsilon_d"] \\ & gd \end{tikzcd} \tag{$**$}$$

The square commutes by the naturality of η and the triangle commutes by one of the triangle identities. But the composition $g\varepsilon_d \circ gfm \circ \eta_c$ is precisely $\varphi\theta(m)$, and thus θ is the right homotopy inverse of φ. The dual argument shows that θ is also the left inverse of φ.

Finally let us conclude by proving $(2) \Rightarrow (3)$. Assume given a unit transformation $\eta : \mathsf{Id}_C \Rightarrow gf$ and using the proof $(1) \Leftrightarrow (2)$ construct a counit transformation $\varepsilon : f \circ g \Rightarrow \mathsf{Id}_D$. Define φ and θ as above ($(3) \Rightarrow (2)$). By the commutativity of the diagram $(*)$ for η and the diagram $(*)$ for ε we deduce that φ and θ are homotopy inverses of each other. Considering $(**)$ as above, and setting $m = \mathsf{Id}_{gd}$ we deduce the triangle identity for g. The dual argument proves the triangle identity for f. ■

It is nice to notice that the above proof is not very different from the classical one that shows that different ways of defining adjunctions coincide. The following proposition can also be proven by lifting the classical argument to the higher categorical setting.

Proposition 24 ([Lur09, 5.2.3.5])**.** *Given an adjunction $f : C \rightleftarrows D : g$ between quasicategories the left adjoint f preserves all colimits which exist in C and the right adjoint g preserves all limits which exists in D.* ■

References

[Lur09] Jacob Lurie. *Higher Topos Theory*. Annals of Mathematics Studies. Princeton University Press, Princeton, NJ, 2009.

[RV15] Emily Riehl and Dominic Verity. The 2-category theory of quasi-categories. *Advances in Mathematics*, 280:549–642, 2015.