Revisiting the mixed-model multi-level just-in-time scheduling problem

JOHN MILTENBURG† and GORDON SINNAMON‡

Morabito and Kraus [1994] propose a modification to the mathematical model of the mixed-model, multi-level just-in-time (JIT) scheduling problem introduced in Miltenburg and Sinnamon [1989]. In this note we relate their proposal to the goals of JIT production and emphasize the importance of using the weights in the original formulation to express management priorities.

Production control in a mixed-model, multi-level production system operating under just-in-time (JIT) principles is executed as illustrated in Figure 1. A schedule is set for assembly of products at the product level, and production at all other levels in the system occurs in response to pull signals. The most common mechanism for generating the pull signals is kanban (Monden [1993]).

The final assembly schedule is set in such a way that the rate of usage of every part in the production system is as close to constant as possible[1]. This leads to the objective; select $x_{i\,j\,k}$, to minimize $\sum_{k=1}^{DT_1} \sum_{j=1}^{4} \sum_{i=1}^{n_j} w_j (x_{i\,j\,k} - XT_{j\,k} r_{i\,j})^2$, where

$x_{i\,j\,k}$ = number of units of part i at level j produced through k stages,
$XT_{i\,j}$ = total production of all parts at level j through k stages,
n_j = number of different parts at level j,
$r_{i\,j} = d_{i\,j}/DT_j$ = fraction of production at level j devoted to part i,
$d_{i\,j}$ = demand for part i at level j,
DT_j = total demand for all parts at level j, and
w_j = weight reflecting the relative importance of variation in usage at level j.

The note by Morabito and Kraus [1994] focuses on the issue of whether the optimal schedule should be constrained by $x_{i\,1\,DT_1} = d_{i\,1}$ so that scheduled production would be exactly equal to demand. Three comments may be made with respect to the appropriateness of this addition to the model.

Comment 1. Variation in usage rates may be more important than meeting demand exactly.

The goal is to determine the final assembly schedule that keeps the usage of each part at each level as constant as possible. Since the product level is included (provided the weight w_1 is not zero) this is usually sufficient to ensure that when production is complete, the right mix of products has been achieved. Note though that the goal is constant rate of usage. The goal is not to produce exactly $d_{i\,1}$ units of each product $i = 1, 2, \ldots$ in exactly DT_1 time periods. JIT (and its pull or kanban control system) is designed for a production environment where production of each part is repetitive. A schedule is set for a week or a

† Schoool of Business, McMaster University, Hamilton, Ontario, Canada L8S 4M4
‡ Department of Mathematics, University of Western Ontario, London, Ontario, Canada N6A 5B7

[1] Jit is a philosophy wherein “wastes” in the production process are identified and removed for the purpose of reducing costs and leadtimes, and improving quality (Ohno [1988]). Scheduling the final assembly line in such a way that usages (and hence production rates) are as close to constant as possible in all parts of the production system facilitates this.

month after which it is changed to accomodate changes in demands. The production of a few units more or less over the entire schedule is not a concern in most instances.

Comment 2. When the model is told to disregard product variation, it will.

If it is important that product demand be met exactly then w_1, the product level weight, should be made large relative to the other weights. Once that is done, minimizing the objective function will tend to force production of the right mix of products. If w_1 is zero (or small relative to the other weights) then variation at the product level will be disregarded and this may lead to situations where product demand is not met. This is the phenomenon exhibited in Morabito and Kraus's example. Example 1 below includes an extreme case of this (Solution 1) and also shows that the model is well behaved when the importance of minimizing product level variation is reflected in the choice of weight values (Solution 2).

Forcing the right mix of subassemblies, etc. will often exert pressure toward the right mix of products as a side effect (see the discussion of Toyota's Goal-Chasing Method in Miltenburg and Sinnamon [1989]) but this effect depends on the bills of material involved. Example 1 shows that this effect cannot be relied upon in all cases.

Example 1. Schedule 2 units each of three products. Each product is made from two subassemblies as follows:

	Product		
Subassembly	1	2	3
1	2	1	0
2	0	1	2

Solution 1. (Disregarding product level variation.) Set $w_1 = 0$ and $w_2 = 1$. The optimal solution is 2, 2, 2, 2, 2, 2, which certainly does not have the right mix of products but maintains a perfectly constant use of subassemblies.

Solution 2. (Comparable regard for variations at the two levels.) Set $w_1 = 1$ and $w_2 = 1$. All optimal schedules (eg. 2, 1, 3, 3, 1, 2) produce the right mix of products.

Example 1 makes it clear that weights have a significant effect and that w_1 should not be set to zero unless extreme variability in product mix can be tolerated.

Comment 3. Adding a new constraint models a different situation.

It may happen that exactly meeting demands is just as important as having a constant rate of product usage. For example, consider the case of a one-time production run with delivery on completion. Although this is not strictly the production environment for which JIT was designed, there are a number of ways of including this in the model.

The constraint suggested by Morabito and Kraus [1994] requiring that demands for products be met exactly could be added. They show that it is easy to adjust the solution algorithm for this constraint. At each cycle k, only those products $i = 1, 2, \ldots$ with $x_{i\,1\,k} < d_{i\,1}$ are eligible to be scheduled.

Another approach is to let the weights w_j vary with i and k (by part and with time)—

that is, replace w_j by $w_{i\,j\,k}$—and set $w_{i\,j\,k} = M$, a very large number, for particular values of i, j and k. This approach provides considerable flexibility to adjust the model for particular situations.

To illustrate this consider Morabito and Kraus's example. They put $w_1 = w_2 = 0$ and add the constraints $x_{i\,1\,DT_1} = d_{i\,1}$ for $i = 1, \ldots, n_1$. This is equivalent to allowing the weights to vary with time and setting $w_{i\,1\,k} = 0$ for $k = 1, \ldots, DT_1 - 1$, $w_{i\,1\,DT_1} = M$, and $w_{i\,2\,k} = 0$ for all k. Or consider the case where a fixed number of one particular subassembly, say item i' at level j', must be produced during k' time periods. (This could occur in an automobile assembly plant when only a limited number of high performance engines are available by a given date.) Only one adjustment to the weights is necessary, $w_{i'\,j'\,k'} = M$. Using weights, rather than adding constraints, is a richer way to fine tune the general model for specific instances of the problem.

Whether additional constraints are added or the weights are allowed to vary, adding checks in Heuristics 1 and 2 is quite appropriate. It is not clear that the modified Heuristics would continue to perform well in this new situation so further testing is recommended.

References

Miltenburg, J. and Sinnamon, G., 1989, Scheduling mixed-model, multi-level, just-in-time production systems. *International Journal of Production Research,* **27,** 1487.

Monden, Y., 1983, *Toyota Production System,* (Norcross, Georgia: Institute of Industrial Engineers Press).

Morabito, M. A. and Kraus, M. E., 1995, A note on "Scheduling mixed-model, multi-level, just-in-time production systems". *International Journal of Production Research,* This issue.

Ohno, T., 1988, *Toyota Production System; Beyond Large-Scale Production,* (Cambridge, Massachusetts: Productivity Press).