Department of Mathematics
Departmental Review
Appendix I: Undergraduate Program
March 2010
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</table>
Chapter 1

The Undergraduate Program

1.1 Undergraduate Program Information

We begin this section with a description of the degree structure at the University of Western Ontario. This is followed by the description of the Modules and Programs that are offered by the Department of Mathematics, and then the calendar copy for the undergraduate courses offered by the department.
THE DEGREE STRUCTURE

There are four possible modules of study which may be entered after First Year:

- Honors Specialization (9.0 or more specified courses)
- Specialization (9.0 or more specified courses)
- Major (6.0 - 7.0 specified courses)
- Minor (4.0 - 5.0 specified courses)

The modular degree structure affords the opportunity to combine various subjects from different Departments and Faculties. The chart below shows how modules can be combined in the three different types of degrees offered. The modules taken must fit within these degree structures. Departments, Schools, Faculties and Affiliated University Colleges may offer some or all of the above modules. See Faculty and Department listings for details. Combinations other than those listed below have not been approved; consequently, they may not be taken. Enrolment in some modules may be limited.

<table>
<thead>
<tr>
<th>DEGREE TYPE</th>
<th>HONORS SPECIALIZATION MODULE</th>
<th>SPECIALIZATION MODULE</th>
<th>MAJOR MODULE</th>
<th>MAJOR MODULE</th>
<th>MINOR MODULE</th>
<th>MINOR MODULE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honors Bachelor</td>
<td>E</td>
<td></td>
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<td>E</td>
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<tr>
<td>Honors Bachelor</td>
<td>E</td>
<td></td>
<td>A</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Honors Bachelor</td>
<td>E</td>
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<td>A</td>
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</tr>
<tr>
<td>Honors Bachelor</td>
<td>E</td>
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<tr>
<td>Bachelor 4 Year</td>
<td>E</td>
<td></td>
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</tr>
<tr>
<td>Bachelor 4 Year</td>
<td>E</td>
<td>E</td>
<td></td>
<td></td>
<td>A</td>
<td></td>
</tr>
</tbody>
</table>
The Honors Bachelor Degree - (20.0 Courses)

15.0 courses after first year, including at least an Honors Specialization or a Double Major, as follows:

- **Honors Specialization module (9.0 or more)**
  This may be combined with a Major module or a Minor module or option(s).
- **Major module (6.0 -7.0) plus a Major module (6.0 - 7.0) plus option(s) (3.0 - 1.0)**
  This combination requires two Major modules.

The Bachelor Degree (Four-Year) - (20.0 Courses)

15.0 courses after first year including at least a Specialization module or a Major module, as follows:

- **Specialization module (9.0 or more)**
  This may be combined with a Major module or a Minor module or option(s).
- **Major module (6.0 -7.0) plus a Major module (6.0 - 7.0) plus option(s) (3.0 - 1.0)**
- **Major module (6.0 -7.0) plus Minor(s) modules(s) or option(s).**
- **Major module (6.0 -7.0) plus options**

The Bachelor Degree (Three-Year) - (15.0 Courses)

10.0 courses after first year including at least a Major module or Double Minors, as follows:

- **Major module (6.0 - 7.0) plus option(s)**
- **Major module (6.0 - 7.0) plus a Minor module (4.0 - 5.0)**
Minor module (4.0 - 5.0) plus a Minor module (4.0 - 5.0) plus option(s) (2.0 - 0)

This combination requires two Minor modules

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Module Combinations and Overlap

Modules in the same discipline normally may not be combined: e.g. an Honors Specialization module in Sociology may not be combined with a Major module or a Minor module in Sociology. However, if a Department offers modules with different titles e.g. Sociology and Criminology, the possibility for combination is at the discretion of the Department and Faculty concerned.

Modules require specific courses, some of which may be common to other modules. Students who wish to combine modules containing the same courses must consult the Department(s) and Faculty concerned to see if such overlap is permitted.

Note: Some degrees limit the number of courses which may be taken in one subject.

Cross Disciplinary

A Cross Disciplinary Major module consists of 6.0 senior courses (numbered 2000 - 4999) approved by the student's Dean's Office. Only available in the Bachelor of Arts Degree (Three-Year). Not offered in the Bachelor of Science degree.

Undeclared Status

Second year students who are taking prerequisite course(s) for a specific module may be registered temporarily in an Undeclared Status within a Three-Year Bachelor Degree only. This status is available only in the Faculties of Arts and Humanities, Science, Social Science and the Affiliated University Colleges. Students progressing into third year must meet the requirements to enter a module. Transfer and Readmitted students who are admitted to an Undeclared Status must consult their Faculty regarding eligibility for specific modules. It is not possible to graduate with a Three-Year Bachelor Undeclared Degree.

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The Honors Bachelor Degree (Four-Year) must include at least an Honors Specialization module or Double Major modules. Registration in an honors degree usually begins in the second year, but admission may be gained in the third or fourth year provided the student has fulfilled the Year 1 principal course requirements and has achieved a minimum cumulative modular average of 70% with no mark less than 60% in the courses of the module and a passing grade in each option. Enrolment in some modules is limited and meeting the minimum requirements does not guarantee that students will be offered enrolment. Students intending to proceed to a four-year Honors degree should consider the degree requirements when selecting courses in first, second and third years. Four-Year Honors degree programs are composed of not fewer than 20.0 successfully completed courses required for the degree. Students admitted with advanced standing to an honors program are required to complete a minimum of 10.0 courses offered by The University of Western Ontario or one of the Affiliated University Colleges.

HONORS SPECIALIZATION MODULE
Admission Requirements

Completion of First Year requirements, including the principal courses specified by the Department. These principal courses must be completed with a minimum average of 70% and a minimum mark of 60% in each. The remaining First Year courses must be completed successfully. Enrolment in some modules may be limited. Higher standards may apply to some modules. Refer to departmental listings.

Progression Requirements

For progression in an Honors Specialization module, a student must earn a minimum cumulative modular average of 70%, a minimum mark of 60% in each course of the module and a passing grade in each option.

Higher progression standards may be required in some modules. Refer to individual Department listings.

In exceptional circumstances, a student who earns a minimum cumulative modular average of 68%, with a minimum mark of 60% in each course of the module and a passing grade in each option, may be permitted to progress by special permission of the Dean on the recommendation of the Department concerned.

Students who fail to meet the progression requirements in an Honors Specialization may be eligible to continue in the Bachelor Degree (Four-Year) in either a Specialization module or a Major module.

Graduation Requirements

Students must meet all graduation requirements for the Honors Bachelor Degree (Four-Year). For complete graduation requirements refer to the "Graduation Regulations" section.

DOUBLE MAJOR MODULES

Admission Requirements

Completion of First Year requirements with at least 3.0 principal courses, including the courses specified for each of two Major modules. If fewer than 3.0 courses are specified, the best additional first year course(s) will be included in the total of 3.0. In some combinations, more than 3.0 specific courses will be required as principal courses. The principal courses must be completed with a minimum average of 70% and a minimum mark of 60% in each. The remaining First Year courses must be completed successfully. Enrolment in some modules may be limited.

Progression Requirements

For progression in an Honors Double Major, a student must earn a minimum cumulative modular average of 70%, a minimum mark of 60% in each course of the module and a passing grade in each option. The modular average for each Major will be calculated separately.

Higher progression standards may be required in some modules. Refer to individual Department listings.

Students who fail to meet the progression requirements of an Honors Double Major may be eligible to continue in the Bachelor Degree (Four-Year) in either a Specialization module or Major module(s).

Graduation Requirements

Students must meet all graduation requirements for the Honors Bachelor Degree (Four-Year). For complete graduation requirements refer to the "Graduation Regulations" section.
The Bachelor Degree (Four-Year) must include at least a Specialization module or at least one Major module. Registration in the Bachelor Degree (Four-Year) usually begins in the second year, but admission may be gained in the third or fourth year. Students intending to proceed to a Bachelor Degree (Four-Year) should consider the degree requirements when selecting courses in first, second and third years. Bachelor Degree (Four-Year) programs are composed of not fewer than 20.0 successfully completed courses required for the degree. Students admitted with advanced standing to a Bachelor Degree (Four-Year) are required to complete a minimum of 10.0 courses offered by The University of Western Ontario or one of the Affiliated University Colleges. Enrolment in some modules may be limited.

**SPECIALIZATION MODULE**

**Admission Requirements**

Completion of First Year requirements, including the principal course(s), specified by the Department with a minimum mark of 60% in each of these course(s). Higher standards may apply to some modules. Refer to Departmental listings. Enrolment in some modules may be limited.

**Progression Requirements**

For progression in a Specialization module, a student must meet the minimum Progression Requirements to continue at the University. See "Progression Requirements" in this Calendar.

Higher progression standards may be required in some modules. Refer to individual Department listings.

**Graduation Requirements**

Students must meet all graduation requirements for the Bachelor Degree (Four-Year). For complete degree requirements, refer to the "Graduation Regulations" section.

**MAJOR MODULE**

**Admission Requirements**

Completion of First Year requirements, including the principal course(s), specified by the Department(s) with a minimum mark of 60% in each. Higher standards may apply to some modules. Refer to Departmental listings. Enrolment in some modules may be limited.

**Progression Requirements**

For progression in a Major module, a student must meet the minimum Progression Requirements to continue at the University. See Progression Requirements Section.

Higher progression standards may be required in some modules.

**Graduation Requirements**

Students must meet all graduation requirements for the Bachelor Degree (Four-Year). For complete graduation requirements, refer to the "Graduation Regulations" section.

The Bachelor Degree (Three-Year) must include at least one Major module or at least two Minor modules. Registration in the Bachelor Degree (Three-Year) usually begins in the second year, but admission may be
gained in the third year. Students intending to proceed to a Bachelor Degree (Three-Year) should consider the degree requirements when selecting courses in first and second years. Bachelor Degree (Three-Year) programs are composed of not fewer than 15.0 successfully completed courses required for the degree. Students admitted with advanced standing to a Bachelor Degree (Three-Year) are required to complete a minimum of 5.0 senior courses (numbered 2000 - 4999) offered by The University of Western Ontario or one of the Affiliated University Colleges. Enrolment in some modules may be limited.

**MAJOR MODULE**

**Admission Requirements**

Completion of First Year requirements, including the principal course(s), specified by the Department with a minimum mark of 60% in each. Enrolment in some modules may be limited.

**Progression Requirements**

For progression in a Major module, a student must meet the minimum Progression Requirements to continue at the University. See “Progression Requirements” section.

Higher progression standards may be required in some modules. Refer to individual Department listings.

**Graduation Requirements**

Students must meet all graduation requirements for the Bachelor Degree (Three-Year). For complete graduation requirements, refer to the “Graduation Regulations” section.

**DOUBLE MINOR MODULES**

**Admission Requirements**

Completion of First Year requirements, including the principal course(s), of each of two Minor modules with a minimum mark of 60% in each principal course. Enrolment in some modules may be limited.

**Progression Requirements**

For progression in Double Minor modules, a student must meet the minimum Progression Requirements to continue at the University. See “Progression Requirements” in this Calendar.

Higher progression standards may be required in some modules. Refer to individual Department listings.

**Graduation Requirements**

Students must meet all graduation requirements for the Bachelor Degree (Three-Year). For complete degree requirements refer to the “Graduation Regulations” section.

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**THE HONORS BACHELOR DEGREE**

<table>
<thead>
<tr>
<th>MODULE</th>
<th>ADMISSION REQUIREMENTS</th>
<th>PROGRESSION REQUIREMENTS</th>
<th>GRADUATION REQUIREMENTS</th>
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<tr>
<td>Honors Specialization</td>
<td>1st Year Requirements</td>
<td>• Minimum cumulative modular average of 70%</td>
<td>On the 20.0 courses counted for graduation:</td>
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<tr>
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<td>• Successful completion of all first</td>
<td></td>
<td>• Minimum</td>
</tr>
<tr>
<td>Double Major</td>
<td>1st Year Requirements</td>
<td>Notes</td>
<td></td>
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<td>--------------</td>
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<tr>
<td></td>
<td>• Successful completion of all first year requirements.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• 70% average on the principal courses for entry to each module.</td>
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</tr>
<tr>
<td></td>
<td>• Minimum mark of 60% in each principal course for entry to each module.</td>
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<tr>
<td></td>
<td><strong>NOTE:</strong> Enrollment in some modules may be limited.</td>
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<tr>
<td></td>
<td><strong>NOTE:</strong> Students who wish to enter this module in a senior year must meet the above requirements and the progression requirements for the module.</td>
<td></td>
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</tr>
</tbody>
</table>

- Minimum mark of 60% in each course of the module.
- Passing grade in each option.

**Note:** Higher progression standards may be required in limited enrolment modules.

- Minimum cumulative modular average of 70% in the Honors Specialization module.
- Minimum mark of 60% in each course of this module.
- Passing grade in each option.
- Minimum overall average of 65% on the 20.0 courses and higher admission standards may be required.

- Minimum cumulative modular average of 60% in any additional Major or Minor module completed.

- For complete graduation requirements refer to ‘Graduation Requirements for the Honors Bachelor Degree.’

- On the 20.0 courses counted for graduation:
  - Minimum cumulative modular average of 70% in each Major module.
  - Minimum mark of 60% in each course of each module.
  - Passing grade in each option.
  - Minimum overall average of 65% on the 20.0 courses.

- For complete graduation
The Bachelor Degree (Four-Year)

<table>
<thead>
<tr>
<th>MODULE</th>
<th>ADMISSION REQUIREMENTS</th>
<th>PROGRESSION REQUIREMENTS</th>
<th>GRADUATION REQUIREMENTS</th>
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</thead>
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<td>Specialization</td>
<td>1st Year Requirements</td>
<td>• Satisfy the Progression Requirements for the University (Level 1 and Level 2).</td>
<td>On the 20.0 courses counted for graduation:</td>
</tr>
<tr>
<td></td>
<td>• Successful completion of all first year requirements.</td>
<td></td>
<td>• Minimum cumulative modular average of 60% in the Specialization module.</td>
</tr>
<tr>
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<td>• Minimum mark of 60% in each principal course for entry to the module.</td>
<td></td>
<td>• Minimum overall average of 60% in the 20.0 courses.</td>
</tr>
<tr>
<td></td>
<td><strong>NOTE:</strong> Enrollment in some modules may be limited.</td>
<td></td>
<td>• Minimum cumulative modular average of 60% in any additional Major or Minor module completed.</td>
</tr>
<tr>
<td></td>
<td><strong>NOTE:</strong> Students who wish to enter this module in a senior year must meet the above requirements and the progression requirements for the module</td>
<td></td>
<td><strong>For complete graduation requirements refer to ‘Graduation Requirements for the Bachelor Degree (Four-Year)’:</strong></td>
</tr>
<tr>
<td>Major</td>
<td>1st Year Requirements</td>
<td>• Satisfy the Progression Requirements for the University (Level 1 and Level 2).</td>
<td>On the 20.0 courses counted for graduation:</td>
</tr>
<tr>
<td></td>
<td>• Successful completion of all first year requirements.</td>
<td></td>
<td>• Minimum cumulative modular average of 60% in the Major module.</td>
</tr>
<tr>
<td></td>
<td>• Minimum mark of 60% in each principal</td>
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</tr>
</tbody>
</table>
NOTE: Enrollment in some modules may be limited.

NOTE: Students who wish to enter this module in a senior year must meet the above requirements and the progression requirements for the module.

NOTE: Higher progression standards may be required in limited enrolment modules.

Minimum overall average of 60% in the 20.0 courses.

Minimum cumulative modular average of 60% in any additional Major or Minor module completed.

For complete graduation requirements refer to ‘Graduation Requirements for the Bachelor Degree (Four-Year)’.

## The Bachelor Degree (Three-Year)

<table>
<thead>
<tr>
<th>MODULE</th>
<th>ADMISSION REQUIREMENTS</th>
<th>PROGRESSION REQUIREMENTS</th>
<th>GRADUATION REQUIREMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major</td>
<td>1st Year Requirements</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Successful completion of all first year requirements.</td>
<td>• Satisfy the Progression Requirements for the University (Level 1 and Level 2).</td>
<td>On the 15.0 courses counted for graduation:</td>
</tr>
<tr>
<td></td>
<td>• Minimum mark of 60% in each principal course for entry to the module.</td>
<td>• Minimum cumulative modular average of 60% in the Major module.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NOTE: Enrollment in some modules may be limited. NOTE: Students who wish to enter this module in a senior year must meet the above requirements and the progression requirements for the module.</td>
<td>NOTE: Higher progression standards may be required in limited enrolment modules.</td>
<td>• Minimum overall average of 60% in the 15.0 courses.</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>• Minimum cumulative modular average of 60% in any additional Major or Minor module completed.</td>
</tr>
<tr>
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<td>For complete graduation requirements refer to ‘Graduation Requirements for Bachelor Degree (Three-Year)’.</td>
</tr>
<tr>
<td>Double Minor</td>
<td>1st Year Requirements</td>
<td>On the 15.0 courses counted for graduation:</td>
<td></td>
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<td>--------------</td>
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<tr>
<td></td>
<td>• Successful completion of all first year requirements.</td>
<td>• Minimum cumulative modular average of 60% in each Minor module.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• 60% average on the principal courses for entry to each module.</td>
<td>• Minimum overall average of 60% in the 15.0 courses</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Minimum mark of 60% in each principal course for entry to each module.</td>
<td>• For complete graduation requirements refer to ‘Graduation Requirements for Bachelor Degree (Three-Year)’.</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** Enrollment in some modules may be limited. **NOTE:** Students who wish to enter this module in a senior year must meet the above requirements and the progression requirements for the module.

**NOTE:** Higher progression standards may be required in limited enrolment modules.

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Major and/or Minor modules may be taken in addition to the essential modules for the degree in accordance with the degree structure outlined previously. Combinations other than those listed in the “Degree Structure” section have not been approved; consequently, they may not be taken. Students graduating with an additional Major or Minor module within their degree must successfully complete the additional module with a minimum average of 60%.

**P**

After graduation with an undergraduate degree, students may complete additional Major and/or Minor modules(s) without completing a second degree. Completion of Post-Degree modules will be recognized only by a notation on the student’s transcript. A Post-Degree module must be in a different subject area, not included in the undergraduate degree. Students must complete the total number of courses required for this additional module. For example, to complete a Post-Degree Minor module, students must successfully complete at least 4.0 courses after graduation. Students must consult the Dean’s Office of the Faculty in which the module is offered for permission to register in a Post-Degree module.

**S**

1. **Honors Bachelor Degree:**

   Students may upgrade to an Honors Bachelor degree in one of two ways:

   1. They may upgrade their Minor, Major or Specialization module(s) in a Bachelor degree (Three or Four-Year) to an Honors Specialization module in an Honors Bachelor degree or they may upgrade their Minor or Major module(s) in a Bachelor degree (Three or Four-Year) to Double Major modules in an Honors Bachelor degree.
They may pursue a new Honors Specialization module in a different subject or Double Major modules in different subjects than those of the previous degree.

All students must complete all outstanding degree requirements in addition to the requirements of the Honors Specialization or Double Major modules chosen.

The following regulations apply to previous University of Western Ontario graduates and graduates of other accredited Universities:

a) Graduates of the University of Western Ontario:

Students who have completed a Bachelor degree (Three-Year) or a Bachelor degree (Four-Year) at The University of Western Ontario may either upgrade to an Honors Bachelor degree or complete the requirements for a different Honors Bachelor degree at the discretion of the department and Faculty concerned. Students must consult the appropriate department(s) or Faculty for assessment of their transcript. They will be considered for a specific Honors Specialization module or Double Major modules in an Honors Bachelor degree if they have met all admission requirements. Meeting minimum admission requirements does not guarantee entrance to an Honors Bachelor degree. The department(s) or Faculty will take into account courses from the first degree and prescribe the courses to be completed. At least 5.0 senior courses must be taken and successfully completed at The University of Western Ontario. Some departments may require more than this number. In addition, the majority of the courses for the Honors Specialization module or for each of the Major modules must be completed through The University of Western Ontario.

b) Graduates of other accredited universities:

Students who have completed a degree from another accredited University must apply for admission to The University of Western Ontario as a Special Student and then, follow the procedure outlined in part "a" above to request a second degree (Honors Bachelor). Students may either upgrade to an Honors Bachelor degree or complete the requirements for a different Honors Bachelor degree at the discretion of the department(s) and Faculty concerned. Graduates of other accredited Universities must also meet The University of Western Ontario residency requirements by taking and successfully completing a minimum of 10.0 senior courses at The University of Western Ontario with honors standards. The majority of the courses for the Honors Specialization module or for each of the Major modules must be completed through The University of Western Ontario.

Notes:

Students applying to upgrade their previous degree conferred under the "old" University of Western Ontario regulations should consult their Faculty regarding permissible upgrades. Students whose previous degree is a professional degree (BA Honors Business Administration, BESc, BFA, BMus, BMusA, BA(HEc), BSc(HEc), BSW(Hons), DDS, LLB or JD, MD, MDiv, MTS) should consult the "Sequential Degree" section for information.

Bachelor Degree (Four-Year)

Students may upgrade to a Bachelor Degree (Four-Year) in one of two ways:

1. They may upgrade their Major or Minor module(s) in a Bachelor degree (Three-Year) to a Specialization module or Double Major modules.
2. They may upgrade to a Bachelor degree (Four-Year) by taking a different Major module, Double Major modules or Specialization module.

All students must complete all outstanding degree requirements for the Bachelor degree (Four-Year) in addition to meeting the specific requirements of the module(s).

The following regulations apply to previous University of Western Ontario graduates and graduates of other accredited Universities:

a) Graduates of The University of Western Ontario:
Students who have completed either a Bachelor degree (Three-Year) or a Bachelor degree (Four-Year) at The University of Western Ontario may apply to upgrade to a Bachelor degree (Four-Year) at the discretion of the department(s) and Faculty concerned. Students must consult the appropriate department(s) or Faculty for assessment of their transcript. At least 5.0 senior courses must be taken and successfully completed at The University of Western Ontario. The majority of the courses for the new modules must be completed through the course offerings of The University of Western Ontario.

b) Graduates of other accredited Universities:
Students who have completed the equivalent of a Bachelor degree (Three-Year or Four-Year) at another accredited University must apply for admission to The University of Western Ontario as a Special Student and then, follow the procedure outlined in part "a" above to request a second Bachelor degree (Four-Year). These students must also meet The University of Western Ontario residency requirements by taking and successfully completing a minimum of 10.0 courses at The University of Western Ontario including all of the required courses for the module(s) chosen.

Notes:

Students applying to upgrade their previous degree conferred under the "old" University of Western Ontario regulations should consult their Faculty regarding permissible upgrades. Students whose previous degree is a professional degree (BA Honors Business Administration, BESc, BFA, BMus, BMusA, BA(HEc), BSc(HEc), BSW(Hons), DDS, LLB or JD, MD, MDiv, MTS) should consult the “Sequential Degree” section for information.

S
Professional Degrees:

BA Honors Business Administration, BESc, BFA, BMus, BMusA, BA(HEc), BSc(FN), BScN, BSc(HEc), BSW(Hons), DDS, LLB or JD, MD, MDiv, MTS

1. Sequential Degree

* Students Who Have Completed a Professional Degree and are Seeking a 3 or 4 Year Bachelor degree or an Honors Bachelor degree.

Students who have been awarded a professional degree by this or another accredited university may be granted advanced standing for a maximum of 10.0 courses toward fulfillment of graduation requirements for a 3 or 4 Year Bachelor degree, or a 4 year Honors Bachelor degree. Consult the Faculty offering the second degree.
Students Who Have Completed a 3 or 4 Year or Honors Bachelor Degree and Are Seeking a Professional Degree

Students who have received a 3 or 4 Year or Honors Bachelor Degree from this or another accredited university may be granted advanced standing toward a professional degree. Consult the Faculty offering the professional degree.

Students Who Have Already Completed a Professional Degree and are Seeking a Second Professional Degree.

Students who have already received a professional degree from this or another accredited university may be granted advanced standing towards the fulfillment of graduation requirements of a second professional degree. Consult the Faculty offering the professional degree.

2. Concurrent Degrees - One Professional and One Bachelor Degree Taken at the Same Time.

Students who are currently registered in a professional degree may apply for permission to register concurrently in a Bachelor degree. Tuition fees applicable to the professional degree will be assessed and primary registration will reflect the Faculty offering the professional degree. (For further fees information, please refer to the Student Financial Services section)

Students must consult the Dean's office of both Faculties for permission to register in, progress in and graduate with a second undergraduate degree with an Honors Specialization, Specialization or Major module.

A complete statement by the Dean of the Faculty offering the 3 or 4 Year or Honors Bachelor degree must be forwarded to the student, with a copy to the Office of the Registrar specifying:

a. The courses that may be credited towards both degrees to a maximum of 10.0, with no more than 5.0 courses from faculties other than Arts and Humanities, Health Sciences (excluding Nursing), Information and Media Studies, Science and Social Science.

b. The number and kind of courses required to complete graduation requirements for the second degree including all senior courses for the Honors Specialization, Specialization or Major.

c. These Concurrent Degree regulations do not apply to Senate approved Combined or Joint degrees.

Note:

In the case of professional degrees for which normal admission requires one or two years of study in a Bachelor degree, the courses taken as part of such a degree must be included among the 10.0 courses double credited towards both degrees.

Students are encouraged to obtain academic counselling from the Dean of each Faculty during the course of their concurrent degree. Students who fail to meet the progression requirements of either degree will be required to withdraw from the concurrent degree.

3. Combined / Joint Degrees

Students who wish to complete Combined or Joint Degrees

Combined or Joint Degrees are Senate approved degrees created by two Faculties where one or both degrees are professional degrees. Normally, these academic options are listed within Departmental or
Faculty degree availability sections. Some examples are as follows:

Bachelor of Engineering Science and Juris Doctor (BESc/JD)
Doctor of Medicine and Doctor of Philosophy (MD/PhD)

Structure of the Degree SR.07-06
HONORS SPECIALIZATION IN MATHEMATICAL SCIENCES

The Honors Specialization in Mathematical Sciences may be used for concurrent degrees in Mathematics and Education.

Admission Requirements
Completion of first-year requirements with no failures. Students must have an average of at least 70% in 3.0 principal courses, including either Calculus 1000A/B or 1100A/B and either Calculus 1301A/B or 1501A/B plus 2.0 additional courses, with no mark in these principal courses below 60%. Students who take Calculus 1301A/B must have a mark of at least 85% in the course. Applied Mathematics 1413 may be substituted for 1.0 Calculus course requirement.

Linear Algebra 1600A/B or Applied Mathematics 1411A/B with a mark of at least 60%, is normally taken in year 1. If not taken in year 1, it must be taken in first term of year 2. Statistical Sciences 1023A/B is recommended.

Module
9.0 courses:

0.5 course from: Applied Mathematics 2811B, Mathematics 2120A/B or the former Mathematics 203b.

0.5 course: Applied Mathematics 2813B.

1.0 course: Calculus 2502A/B, 2503A/B.

0.5 course: Applied Mathematics 2402A or the former Differential Equations 2402A.

0.5 course: Mathematics 2124A/B.

1.0 course: Statistical Sciences 2857A/B or the former 2657A, Statistical Sciences 2858A/B.

0.5 course from: Statistical Sciences 3657A/B or the former 3652A/B.

0.5 course from: Applied Mathematics 3811A/B or Mathematics 3124A/B.

1.0 course from: Group A: Courses Emphasizing Proofs.

• Applied Mathematics 3815A/B or 4615F/G.
• Mathematics 2122A/B, 2155A/B, 2251F/G, 3120A/B, 3122A/B, 3150A/B, 3154A/B, 4123A/B or the former 308a/b.
• Statistical Sciences 3858A/B, 4654A/B.

1.0 course from: Group B: Applications.

• Actuarial Science 2553A/B, 2555A/B.
• Mathematics 2156A/B, 3152A/B.
• Statistical Sciences 3843A/B, 3850F/G, 3859A/B, 4521F/G, 4846A/B.
HONORS SPECIALIZATION IN MATHEMATICS

Admission Requirements
Completion of first-year requirements with no failures. Students must have an average of at least 70% in 3.0 principal courses, including:

- Calculus 1000A/B or 1100A/B;
- Calculus 1501A/B (or Calculus 1301A/B with a mark of at least 85%);
- plus 2.0 additional courses, with no mark in these principal courses below 60%. Linear Algebra 1600A/B and Mathematics 1120A/B, if taken in first year, will count toward the 3.0 principal courses.

Linear Algebra 1600A/B and Mathematics 1120A/B are recommended.

**Note:** Linear Algebra 1600A/B with a minimum mark of 60% or Mathematics 1120A/B with a minimum mark of 70% must be completed prior to Mathematics 2120A/B.

Module
9.0 courses:

- **1.0 course**: Calculus 2502A/B, 2503A/B.
- **5.0 courses**: Mathematics 2120A/B or the former 203b, Mathematics 2121A/B or 2155A/B, 2122A/B, 2123A/B, 2124A/B, 3020A/B, 3120A/B, 3122A/B, 3123A/B, 3124A/B.
- **2.0 additional courses** from Mathematics 2121A/B, 2155A/B, 2156A/B, or any Mathematics course at the 3000 level or above.
- **1.0 additional course** in Mathematics at the 4000 level.

**Note:** Those students who plan to apply for graduate studies in Mathematics should take Mathematics 3132B, 4120A/B, 4122A/B, 4123A/B, and at least one of Mathematics 4151A/B, 4152A/B, 4153A/B or 4156A/B.
HONORS SPECIALIZATION IN MATHEMATICS IN SOCIETY

Admission Requirements
Completion of first-year requirements with no failures. Students must have an average of at least 70% in 3.0 principal courses, including Calculus 1000A/B or 1100A/B, Calculus 1501A/B (or Calculus 1301A/B with a mark of at least 85%), plus 2.0 additional courses, with no mark in these principal courses below 60%. Linear Algebra 1600A/B and Mathematics 1120A/B, if taken in first year, will count toward the 3.0 principal courses.

Linear Algebra 1600A/B and Mathematics 1120A/B are recommended.

Note: Linear Algebra 1600A/B with a minimum mark of 60% or Mathematics 1120A/B with a minimum mark of 70% must be completed prior to Mathematics 2120A/B.

Module
9.0 courses:

4.0 courses: Calculus 2502A/B, 2503A/B, Mathematics 2120A/B, 2122A/B, 2155A/B, 2212A/B, 3020A/B, 3150A/B.


2.5 courses from: Actuarial Sciences, Applied Mathematics, Computer Science, Mathematics, or Statistics courses, at the 2100 level or above.

Note: Students intending to pursue graduate studies in Pure Mathematics should take the Honors Specialization in Mathematics module.
MAJOR IN MATHEMATICS

Admission Requirements
Completion of first-year requirements. Students must have an average of at least 70% in 3.0 principal courses, including:

(Calculus 1000A/B or 1100A/B) and (Calculus 1501A/B (or Calculus 1301A/B with a mark of at least 85%)), plus 2.0 additional courses, with no mark in these principal courses below 60%. Linear Algebra 1600A/B and Mathematics 1120A/B, if taken in first year, will count toward the 3.0 principal courses.

Linear Algebra 1600A/B and Mathematics 1120A/B are recommended.

Note: Linear Algebra 1600A/B with a minimum mark of 60% or Mathematics 1120A/B with a minimum mark of 70% must be completed prior to Mathematics 2120A/B.

Module
6.0 courses:

1.0 course: Calculus 2502A/B, 2503A/B.
2.0 courses: Mathematics 2120A/B or the former 203b, Mathematics 2122A/B, 2123A/B, 2124A/B.
0.5 course from: Mathematics 2121A/B or 2155A/B.
2.5 additional courses from: Mathematics 2156A/B and Mathematics courses at the 3000 level or above.
SPECIALIZATION IN MATHEMATICS

Admission Requirements
Completion of first-year requirements, including either Calculus 1000A/B or 1100A/B with a mark of at least 60%, and either Calculus 1501A/B with a mark of at least 60% (recommended) or Calculus 1301A/B with a mark of at least 85%.

Linear Algebra 1600A/B and Mathematics 1120A/B are recommended.

Note: Students who plan to take Mathematics 2120A/B must first complete either Linear Algebra 1600A/B with a minimum mark of 60% or Mathematics 1120A/B with a minimum mark of 70%.

Students who plan to take Mathematics 2211B must first complete either Linear Algebra 1600A/B (with a minimum mark of 50%) or Mathematics 1120A/B with a minimum mark of 70%.

Module
9.0 courses:

- 0.5 course from: Calculus 2502A/B recommended or Calculus 2302A/B.
- 0.5 course from: Calculus 2503A/B recommended or Calculus 2303A/B.
- 0.5 course from: Mathematics 2120A/B or 2211A/B, or the former 203b.
- 1.5 courses: Mathematics 2122A/B, 2123A/B, 2124A/B.
- 0.5 course from: Mathematics 2121A/B or 2155A/B.
- 1.0 course from: Mathematics 3020A/B and 3120A/B or Mathematics 2290.
- 0.5 course from: Mathematics 3123A/B; or Applied Mathematics 2402A or the former Differential Equations 2402A.
- 0.5 course from: Mathematics 3124A/B or 2212A/B.
- 3.5 additional Mathematics courses at the 2000 level or above. (Up to 2.0 courses may be substituted with courses offered by the Departments of Applied Mathematics or Statistical and Actuarial Sciences with the approval of the Department of Mathematics).
SPECIALIZATION IN MATHEMATICS IN SOCIETY

Admission Requirements

Completion of first-year requirements including Calculus 1000A/B or 1100A/B with a mark of at least 60%, and either Calculus 1501A/B with a mark of at least 60% (recommended) or Calculus 1301A/B with a mark of at least 85%.

Linear Algebra 1600A/B and Mathematics 1120A/B are recommended.

Note: Linear Algebra 1600A/B with a minimum mark of 60%, or Mathematics 1120A/B with a minimum mark of 70%, must be completed prior to Mathematics 2120A/B.

Module

9.0 courses:

2.0 courses: Calculus 2302A/B or 2502A/B, Calculus 2303A/B or 2503A/B, Mathematics 2120A/B, 2122A/B.


4.0 courses from: Actuarial Sciences, Applied Mathematics, Computer Science, Mathematics, or Statistics courses, at the 2100 level or above.
CONCURRENT MATHEMATICS AND EDUCATION PROGRAM - BSc/BEd

Note: as of September, 2009, this program is no longer being offered.

This five-year program is designed to prepare students to teach Mathematics at the intermediate-senior level. Graduates will earn a BSc or Honors BSc as well as a BEd. Entry into the program is after completion of first year. Students who wish to transfer into this program after their second year must consult with the Associate Dean, Undergraduate and Preservice Programs in the Faculty of Education, and a representative for the program from the Faculty of Science.

The program consists of three parts. First is a Specialization or Honors Specialization offered by the Faculty of Science, chosen from the following list:

- Specialization in Mathematics in Society
- Honors Specialization in Mathematics in Society
- Honors Specialization in Mathematical Sciences (see Applied Mathematics)
- Honors Specialization in Mathematics

Second is the Minor for Concurrent Mathematics and Education Programs module, offered by the Faculty of Education. These two parts form the BSc. The third part is the BEd degree offered by the Faculty of Education.

Admission Requirements

Students must meet module admission requirements for the chosen Science module and for the Minor for Concurrent Programs. In addition students must be accepted to the Faculty of Education based on their first-year performance and on the completion of a written Experience Profile. The program is of limited size and entry may be on a competitive basis.
1.3 Undergraduate Course Calendar Copy

Calculus 1000A/B - Calculus I

Antirequisite(s): Calculus 1100A/B, Applied Mathematics 1413, the former Mathematics 030.

Prerequisite(s): One or more of Ontario Secondary School MCV4U, Mathematics 0110A/B, or the former Ontario Secondary School MCB4U.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 4 lecture hours, 0.5 course.

Calculus 1100A/B - Calculus I with Fundamentals

Antirequisite(s): Calculus 1000A/B, Applied Mathematics 1413, the former Mathematics 030.

Prerequisite(s): One or more of Ontario Secondary School MHF4U, MCV4U, Mathematics 0110A/B, or the former Ontario Secondary School MCB4U.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: Note: This course is intended for students with no previous experience of

Calculus 1301A/B - Calculus II
For students requiring the equivalent of a full course in calculus at a less rigorous level than Calculus 1501A/B. Integration by parts, partial fractions, integral tables, geometric series, harmonic series, Taylor series with applications, arc length of parametric and polar curves, first order linear and separable differential equations with applications.

Antirequisite(s): Calculus 1501A/B, Applied Mathematics 1413.

Prerequisite(s): A minimum mark of 55% in one of Calculus 1000A/B or 1100A/B.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 4 lecture hours, 0.5 course.
Calculus 1501A/B - Calculus II for Mathematical and Physical Sciences

Students who intend to pursue a degree in Actuarial Science, Applied Mathematics, Astronomy, Mathematics, Physics, or Statistics should take this course. Techniques of integration; The Mean Value Theorem and its consequences; series, Taylor series with applications; parametric and polar curves with applications; first order linear and separable differential equations with applications.

**Antirequisite(s):** [Calculus 1301A/B](#), [Applied Mathematics 1413](#).

**Prerequisite(s):** A minimum mark of 60% in one of [Calculus 1000A/B](#) or 1100A/B.

**Corequisite(s):**

**Extra Information:** 4 lecture hours, 0.5 course.

Calculus 2502A/B - Advanced Calculus I

Differential calculus of functions of several variables: level curves and surfaces; limits; continuity; partial derivatives; total differentials; Jacobian matrix; chain rule; implicit functions; inverse functions; curvilinear coordinates; derivatives; the Laplacian; Taylor Series; extrema; Lagrange multipliers; vector and scalar fields; divergence and curl.

**Antirequisite(s):** [Calculus 2302A/B](#), or the former Applied Mathematics 290a.

**Prerequisite(s):** A minimum mark of 60% in [Calculus 1501A/B](#) or Applied Mathematics 1413, or [Calculus 1301A/B](#) with a mark of at least 85%.

**Corequisite(s):**

**Pre-or Corequisite(s):** [Linear Algebra 1600A/B](#) or the former Mathematics 202a.

**Extra Information:** 3 lecture hours, 0.5 course.

Linear Algebra 1600A/B - Linear Algebra I

Properties and applications of vectors; matrix algebra; solving systems of linear equations; determinants; vector spaces; orthogonality; eigenvalues and eigenvectors.

**Antirequisite(s):** [Applied Mathematics 1411A/B](#), 2811B.

**Prerequisite(s):** One or more of Ontario Secondary School MCV4U, the former Ontario Secondary School MGA4U, Mathematics 1229A/B, the former Mathematics 017a/b, [Calculus 1100A/B](#), or [Calculus 1000A/B](#) taken as a pre- or co-requisite.

**Corequisite(s):**

**Pre-or Corequisite(s):**

**Extra Information:** 3 lecture hours, 1 laboratory hour, 0.5 course.
Mathematics 0110A/B - Introductory Calculus
Introduction to differential calculus including limits, continuity, definition of derivative, rules for differentiation, implicit differentiation, velocity, acceleration, related rates, maxima and minima, exponential functions, logarithmic functions, differentiation of exponential and logarithmic functions, curve sketching.

Antirequisite(s): Mathematics 1225A/B, Calculus 1000A/B, Calculus 1100A/B, Applied Mathematics 1413, the former Mathematics 030.
Prerequisite(s): One or more of Ontario Secondary School MCF3M, MCR3U, or equivalent.
Corequisite(s):

Extra Information: 4 lecture hours, 0.5 course.

Mathematics 1120A/B - Fundamental Concepts in Mathematics
Primarily for students interested in pursuing a degree in one of the mathematical sciences. Logic, set theory, relations, functions and operations, careful study of the integers, discussion of the real and complex numbers, polynomials, and infinite sets.

Antirequisite(s): Mathematics 2155A/B.
Prerequisite(s): One or more of Ontario Secondary School MCV4U, the former Ontario Secondary School MGA4U, Linear Algebra 1600A/B, or the former Mathematics 017a/b.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 4 lecture hours, 0.5 course.

Mathematics 1120A/B - Fundamental Concepts in Mathematics
Primarily for students interested in pursuing a degree in one of the mathematical sciences. Logic, set theory, relations, functions and operations, careful study of the integers, discussion of the real and complex numbers, polynomials, and infinite sets.

Antirequisite(s): Mathematics 2155A/B.
Prerequisite(s): One or more of Ontario Secondary School MCV4U, the former Ontario Secondary School MGA4U, Linear Algebra 1600A/B, or the former Mathematics 017a/b.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 4 lecture hours, 0.5 course.
Mathematics 1228A/B - Methods of Finite Mathematics
Permutations and combinations; probability theory. This course is intended primarily for students in the Social Sciences, but may meet minimum requirements for some Biological or Basic Medical Sciences modules.
**Antirequisite(s):** Mathematics 2124A/B, 2155A/B, Statistical Sciences 2035, 2141A/B, 2857A/B, the former Mathematics 031, the former Statistical Sciences 2657A.
**Prerequisite(s):** One or more of Ontario Secondary School MCV4U, MHF4U, MDM4U, Mathematics 0110A/B, 1225A/B, 1229A/B, the former Mathematics 017a/b, the former Ontario Secondary School MGA4U, MCB4U.
**Corequisite(s):**
**Pre-or Corequisite(s):**
**Extra Information:** 3 lecture hours, 0.5 course.

Mathematics 1229A/B - Methods of Matrix Algebra
Matrix algebra including vectors and matrices, linear equations, determinants. This course is intended primarily for students in the Social Sciences, but may meet minimum requirements for some Biological or Basic Medical Sciences modules.
**Antirequisite(s):** Applied Mathematics 1411A/B, 2811B, Linear Algebra 1600A/B, Mathematics 2120A/B, 2155A/B, 2211A/B, the former Mathematics 030, 031, 203b.
**Prerequisite(s):** One or more of Ontario Secondary School MCF3M, MCR3U, or equivalent.
**Corequisite(s):**
**Pre-or Corequisite(s):**
**Extra Information:** 3 lecture hours, 0.5 course.

Mathematics 2120A/B - Intermediate Linear Algebra I
A rigorous development of lines and planes in $\mathbb{R}^n$; linear transformations and abstract vector spaces. Determinants and an introduction to diagonalization and its applications including the characteristic polynomials, eigenvalues and eigenvectors.
**Antirequisite(s):** Mathematics 2211A/B and the former Mathematics 203b.
**Prerequisite(s):** Linear Algebra 1600A/B with a minimum mark of 60% or Mathematics 1120A/B with a minimum mark of 70% or permission of the Mathematics Department.
**Corequisite(s):**
**Pre-or Corequisite(s):**
**Extra Information:** 3 lecture hours, 0.5 course.
Mathematics 2121A/B - Intermediate Linear Algebra II
A continuation of the material of Mathematics 2120A/B including properties of complex numbers and the principal axis theorem; singular value decomposition; linear groups; similarity; Jordan canonical form; Cayley-Hamilton theorem; bilinear forms; Sylvester's theorem.

Antirequisite(s):
Prerequisite(s): Mathematics 2120A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

Mathematics 2122A/B - Real Analysis I
A rigorous introduction to analysis on the real line, primarily for honors students. Sets, functions, natural numbers, Axioms for the real numbers, Completeness and its consequences, Sequences and limits, Continuous and differentiable functions, The Mean Value Theorem.

Antirequisite(s):
Prerequisite(s): Calculus 1501A/B or Applied Mathematics 1413, with a minimum mark of 60%, or Calculus 1301A/B with a minimum mark of 85%.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 4 lecture hours, 0.5 course.

Mathematics 2123A/B - Real Analysis II

Antirequisite(s): The former Mathematics 306a/b.
Prerequisite(s): Mathematics 2122A/B with a minimum mark of 60%, or permission of the Department.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
Mathematics 2124A/B - Introduction to Mathematical Problems

Primarily for Mathematics students, but will interest other students with ability in and curiosity about mathematics in the modern world as well as in the past. Stresses development of students' abilities to solve problems and construct proofs. Topics will be selected from: counting, recurrence, induction; number theory; graph theory; parity, symmetry; geometry.

**Antirequisite(s):**

**Prerequisite(s):** Calculus 1501A/B or Applied Mathematics 1413, with a minimum mark of 60%, or Calculus 1301A/B with a minimum mark of 85% or permission of the instructor.

**Corequisite(s):**

**Extra Information:** 3 lecture hours, 0.5 course.

Mathematics 2155A/B - Discrete Structures I

This course provides an introduction to logical reasoning and proofs. Topics include sets, counting (permutations and combinations), mathematical induction, relations and functions, partial order relations, equivalence relations, groups and applications to error-correcting codes.

**Antirequisite(s):** Software Engineering 2251A/B

**Prerequisite(s):** 1.0 course from: Mathematics 1120A/B, Applied Mathematics 1413, Calculus 1000A/B or 1100A/B, Calculus 1301A/B or 1501A/B, Linear Algebra 1600A/B, or the former Mathematics 030 (in each case with a minimum mark of 60%).

**Corequisite(s):**

**Extra Information:** 4 lecture hours, 0.5 course.

Mathematics 2156A/B - Discrete Structures II

This course continues the development of logical reasoning and proofs begun in Mathematics 2155A/B. Topics include elementary number theory (gcd, lcm, Euclidean algorithm, congruences, Chinese remainder theorem) and graph theory (connectedness, complete, regular and bipartite graphs; trees and spanning trees, Eulerian and Hamiltonian graphs, planar graphs; vertex, face and edge colouring; chromatic polynomials).

**Antirequisite(s):**

**Prerequisite(s):** Mathematics 2155A/B.

**Corequisite(s):**

**Extra Information:** 4 lecture hours, 0.5 course.
Mathematics 2211A/B - Linear Algebra
Linear transformations, matrix representation, rank, change of basis, eigenvalues and eigenvectors, inner product spaces, quadratic forms and conic sections. Emphasis on problem-solving rather than theoretical development. Cannot be taken for credit by students in honors Mathematics programs.
Antirequisite(s): Applied Mathematics 2811B, Mathematics 2120A/B, the former Mathematics 203b.
Prerequisite(s): Linear Algebra 1600A/B or Mathematics 1120A/B with a minimum mark of 70%, or the former Applied Mathematics 202a or 212a.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

Mathematics 2212A/B - Complex Variables
Complex numbers, Cauchy-Riemann equations, elementary functions, integrals, Cauchy’s theorem and integral formula and applications, Taylor and Laurent expansions, isolated singularities, residue theorem and applications. Cannot be taken for credit by students in honors Mathematics programs.
Antirequisite(s): Mathematics 3124A/B, Applied Mathematics 3811A/B, the former Mathematics 306a/b.
Prerequisite(s): Calculus 2302A/B or Calculus 2502A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

Mathematics 2251F/G - Conceptual Development of Mathematics
A survey of some important basic concepts of mathematics in a historical setting, and in relation to the broader history of ideas. Topics may include: the evolution of the number concept, the development of geometry, Zeno's paradoxes.
Antirequisite(s): Philosophy 2251F/G.
Prerequisite(s): 1.0 course of university level Mathematics.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
Mathematics 2290 - Algebra
An introduction to abstract algebra, with principal emphasis on the structure of groups, rings, integral domains and fields. Cannot be taken for credit by students in honors Mathematics programs.
Antirequisite(s): Mathematics 3120A/B.
Prerequisite(s): 0.5 course from: Linear Algebra 1600A/B, Mathematics 2120A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 1.0 course.

Mathematics 2291 - Elementary Theory of Numbers
Euclidean algorithm, congruences, indices, continued fractions, Gaussian integers, partitions and Diophantine equations.
Antirequisite(s): Mathematics 3150A/B.
Prerequisite(s): 1.0 course from: Calculus 1000A/B, 1100A/B, 1301A/B or 1501A/B, Applied Mathematics 1413, Mathematics 1120A/B, 1225A/B, 1228A/B, 1229A/B, Linear Algebra 1600A/B, the former Mathematics 030, 031.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 1.0 course.

Mathematics 2292 - Synthetic Geometry
Groups of transformations of the Euclidean plane, inversion, the projective plane.
Antirequisite(s): Mathematics 4153A/B, the former Mathematics 319a/b.
Prerequisite(s): 1.0 course from: Calculus 1000A/B, 1100A/B, 1301A/B or 1501A/B, Applied Mathematics 1413, Mathematics 1120A/B, 1225A/B, 1228A/B, 1229A/B, Linear Algebra 1600A/B, the former Mathematics 030, 031.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 1.0 course.
Mathematics 2293 - Elementary Operations Research with Applications
Linear programming, basic probability and statistical distributions, networks, decision
analysis, utility, game theory, inventory analysis, queuing theory, simulation, Markovian
decision model, forecasting. Cannot be taken for credit by students in honors Mathematics
programs.
Antirequisite(s): Applied Mathematics 3817A/B, Statistical Sciences 4654A/B, the former
Statistical Sciences 236, 4737A/B.
Prerequisite(s): 1.0 course from: Calculus 1000A/B, 1100A/B, 1301A/B or 1501A/B, Applied
Mathematics 1413, Mathematics 1120A/B, 1225A/B, 1228A/B, 1229A/B, Linear Algebra
1600A/B, the former Mathematics 030, 031. If Mathematics 1228A/B or the former
Mathematics 031 is not taken, one of the following is also required, either as a prerequisite or a
fall term co-requisite: Economics 2122A/B, 2222A/B, Statistical Sciences 2035, 2141A/B.
Corequisite(s):
Extra Information: 3 lecture hours, 1.0 course.

Mathematics 3020A/B - Introduction to Abstract Algebra
Sets and binary operations, commutativity, associativity, distributivity, groups and subgroups,
cyclic groups, permutation groups, cosets, Lagrange’s theorem, normal subgroups, quotient
groups, first isomorphism theorem, rings, integral domains, fields, polynomial rings, unique
factorization of polynomials over a field, finite fields.
Antirequisite(s):
Prerequisite(s): Mathematics 2120A/B or Applied Mathematics 2811B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

Mathematics 3120A/B - Group Theory
An introduction to the theory of groups: cyclic, dihedral, symmetric, alternating; subgroups,
quotient groups, homomorphisms, cosets, Lagrange’s theorem, isomorphism theorems; group
actions, class equation, p-groups, Sylow theorems; direct and semidirect products, wreath
products, finite abelian groups; Jordan-Hölder theorem, commutator subgroup, solvable and
nilpotent groups; free groups, generators and relations.
Antirequisite(s):
Prerequisite(s): Mathematics 3020A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
Mathematics 3122A/B - Metric Space Topology
An introduction to the theory of metric spaces with emphasis on the point-set topology of Euclidean n-space, including convergence, compactness, completeness, continuity, uniform continuity, homeomorphism, equivalence of metrics, connectedness, path-connectedness, fixed-point theorem for contractions, separability, complete normality, product spaces, category.

Antirequisite(s):

Prerequisite(s): Either Mathematics 2122A/B or Mathematics 2123A/B, each with a minimum mark of 60%.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.

Mathematics 3123A/B - Differential Equations

Antirequisite(s):

Prerequisite(s): 2.0 courses: Calculus 2503A/B; Mathematics 2123A/B or the former Mathematics 306a/b; Mathematics 3122A/B; Mathematics 2120A/B; or Applied Mathematics 2811B or the former Mathematics 203b.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.

Mathematics 3124A/B - Complex Analysis I
The Cauchy-Riemann equations, elementary functions, branches of the logarithm and argument, Cauchy's integral theorem and formula, winding number, Liouville's theorem and the fundamental theorem of algebra, the identity theorem, the maximum modulus theorem, Taylor and Laurent expansions, isolated singularities, the residue theorem and applications, the argument principle and applications.

Antirequisite(s): Applied Mathematics 3811A/B.

Prerequisite(s): Mathematics 2123A/B or the former Mathematics 306a/b.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
Mathematics 3132B - General Topology
Topological spaces, operations on subsets (e.g. closure), neighbourhoods, bases, subspaces, quotient spaces, product spaces, connectedness, compactness, countability and separation axioms, function spaces.

Antirequisite(s): The former Mathematics 4121A.
Prerequisite(s): Mathematics 3122A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

Mathematics 3150A/B - Elementary Number Theory I
Divisibility, primes, congruences, theorems of Fermat and Wilson, Chinese remainder theorem, quadratic reciprocity, some functions of number theory, diophantine equations, simple continued fractions.

Antirequisite(s): Mathematics 2291.
Prerequisite(s): 1.0 course in Mathematics, Applied Mathematics, Calculus, or Differential Equations at the 2100 level or higher.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

Mathematics 3151A/B - Elementary Number Theory II
Arithmetic functions, perfect numbers, the Möbius inversion formula, introduction to Dirichlet series and the Riemann zeta function, some methods of combinatorial number theory, primitive roots and their relationship with quadratic reciprocity, the Gaussian integers, sums of squares and Minkowski's theorem, square and triangular numbers, Pell's equation, introduction to elliptic curves.

Antirequisite(s): Mathematics 2291.
Prerequisite(s): Mathematics 3150A/B or Mathematics 2156A/B.
Corequisite(s):
Pre-or Corequisite(s):
Mathematics 3152A/B - Combinatorial Mathematics
Enumeration, recursion and generating functions, linear programming, Latin squares, block
designs, binary codes, groups of symmetries, orbits, and counting.
**Antirequisite(s):**
**Prerequisite(s):** 0.5 course from: Mathematics 2120A/B, 2156A/B, 2211A/B, Applied
Mathematics 2811B or the former Mathematics 203b, or permission of the Department.
**Corequisite(s):**
**Pre-or Corequisite(s):**
**Extra Information:** 3 lecture hours, 0.5 course.

Mathematics 3153A/B - Discrete Optimization
**Antirequisite(s):**
**Prerequisite(s):** One of: Mathematics 2156A/B, 3152A/B, the former Statistical Sciences 236, or permission of the
Department.
**Corequisite(s):**
**Pre-or Corequisite(s):**
**Extra Information:** 3 lecture hours, 0.5 course.

Mathematics 3154A/B - Introduction to Algebraic Curves
Geometry of algebraic curves over the rational, real and complex fields. Classification of
affine conics, singularities, intersection numbers, tangents, projective algebraic curves,
multiplicity of points, flexes. Some discussion of cubic curves.
**Antirequisite(s):** Mathematics 2292 and the former Mathematics 319a/b
**Prerequisite(s):** Linear Algebra 1600A/B or Mathematics 2120A/B; Mathematics 2121A/B,
2122A/B, 2124A/B or 2155A/B; an additional 0.5 course in Mathematics, Applied
Mathematics, Calculus, or Differential Equations at the 2100 level or above.
**Corequisite(s):**
**Pre-or Corequisite(s):**
**Extra Information:** 3 lecture hours, 0.5 course.
Mathematics 3958A/B - Special Topics in Mathematics

Antirequisite(s):

Prerequisite(s): Permission of the Department.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.

Mathematics 3959A/B - Special Topics in Mathematics

Antirequisite(s):

Prerequisite(s): Permission of the Department.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.

Mathematics 4120A/B - Field Theory

Automorphisms of fields, separable and normal extensions, splitting fields, fundamental theorem of Galois theory, primitive elements, Lagrange's theorem. Finite fields and their Galois groups, cyclotomic extensions and polynomials, applications of Galois theory to geometric constructions and solution of algebraic equations.

Antirequisite(s):

Prerequisite(s): Mathematics 4123A/B, the former Mathematics 308a/b.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
Mathematics 4122A/B - Lebesgue Integration and Fourier Series

Lebesgue measure, measurable sets and functions, approximation theorems, the Lebesgue integral, comparison with the Riemann integral, the basic convergence theorems and continuity properties, the space $L_2$, the Riesz-Fischer theorem and completeness of the trigonometric system, pointwise convergence of Fourier series, Fejér's theorem.

**Antirequisite(s):**
**Prerequisite(s):** Mathematics 3122A/B.
**Corequisite(s):**
**Pre-or Corequisite(s):**
**Extra Information:** 3 lecture hours, 0.5 course.

Mathematics 4123A/B - Rings and Modules

Commutative rings, ring homomorphisms and quotient rings, ideals, rings of fractions, the Chinese remainder theorem; Euclidean domains, principal ideal domains, unique factorization domains; polynomial rings over fields; modules, direct sums of modules, free modules; modules over a principal ideal domain, the rational canonical form, the Jordan canonical form.

**Antirequisite(s):** The former Mathematics 308a/b.
**Prerequisite(s):** Mathematics 3120A/B.
**Corequisite(s):**
**Pre-or Corequisite(s):**
**Extra Information:** 3 lecture hours, 0.5 course.

Mathematics 4151A/B - Algebraic Number Theory

Algebraic numbers, cyclotomic fields, low dimensional Galois cohomology, Brauer groups, quadratic forms, local and global class fields, class field theory, Galois group representations, modular forms and elliptic curves, zeta function and L-series.

**Antirequisite(s):**
**Prerequisite(s):** Mathematics 4120A/B; Mathematics 3151A/B strongly recommended but not required.
**Corequisite(s):**
**Pre-or Corequisite(s):**
**Extra Information:** 3 lecture hours, 0.5 course.
Mathematics 4153A/B - Algebraic Geometry
Affine and projective varieties, coordinate rings and function fields, birational correspondences, sheaves, dimension theory, regularity.

Antirequisite(s):

Prerequisite(s): Mathematics 4120A/B; Mathematics 3154A/B is recommended but not required.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.

Mathematics 4155A/B - Multivariable Calculus
Review of differentiability in Euclidean space, inverse and implicit function theorems, integration in Euclidean space, Fubini's theorem, partitions of unity, change of variable, multilinear functions, tensor and wedge product, vector fields, differential forms, Poincaré's lemma, Stokes' theorem, manifolds, fields and forms on manifolds, Stokes' theorem on manifolds.

Antirequisite(s):

Prerequisite(s): Calculus 2503A/B, and Mathematics 3122A/B.

Corequisite(s):

Pre-or Corequisite(s):

Mathematics 4157A/B - Complex Variables III
Entire and meromorphic functions, infinite products, canonical products, the Weierstrass factorization and Mittag-Leffler theorems, the Hadamard factorization theorem; simply periodic and doubly periodic functions, elliptic functions; the Picard theorems (with Schottky's, Montel's, and Landau's theorems); the prime number theorem (with the Gamma and Riemann Zeta functions).

Antirequisite(s):

Prerequisite(s): Mathematics 4156A/B or Mathematics 3124A/B with the permission of the Department.

Corequisite(s):
Mathematics 4160A/B - Ordinary Differential Equations
Laplace transforms and their application to solving differential equations. Sturm-Liouville systems; eigenvalue problems, expansions, Fourier series, autonomous systems; linear and non-linear problems, types of critical points, stability.
Antirequisite(s):
Prerequisite(s): Mathematics 3123A/B; or Applied Mathematics 2402A or the former Differential Equations 2402A.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

Mathematics 4161A/B - Linear Ordinary Differential Equations
First order vector systems and nth order single equations; adjoint systems and boundary value problems; Green's functions and self adjoint eigenvalue problems; expansion theory and spectral decomposition.
Antirequisite(s):
Prerequisite(s): Mathematics 3123A/B; or Applied Mathematics 2402A or the former Differential Equations 2402A.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

Mathematics 4958A/B - Special Topics in Mathematics
Antirequisite(s):
Prerequisite(s): Permission of the Department.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
Mathematics 4959A/B - Special Topics in Mathematics

Antirequisite(s): 
Prerequisite(s): Permission of the Department.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

1.4 Undergraduate Course Information

For each undergraduate course offered by the Department of Mathematics, we provide the current calendar copy (for the reader’s convenience, as the complete collection of course calendar copies is available in the preceding section), a recent course outline, a recent final exam, and recent evaluations for the course.
1.4.1 Calculus 1000 and 1100

Calculus 1000A/B - Calculus I
Antirequisite(s): Calculus 1100A/B, Applied Mathematics 1413, the former Mathematics 030.
Prerequisite(s): One or more of Ontario Secondary School MCV4U, Mathematics 0110A/B, or the former Ontario Secondary School MCB4U.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 4 lecture hours, 0.5 course.

Calculus 1100A/B - Calculus I with Fundamentals
Antirequisite(s): Calculus 1000A/B, Applied Mathematics 1413, the former Mathematics 030.
Prerequisite(s): One or more of Ontario Secondary School MHF4U, MCV4U, Mathematics 0110A/B, or the former Ontario Secondary School MCB4U.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: Note: This course is intended for students with no previous experience of

Department of Mathematics:  R. Bryan (King’s), E. Coskun, S. Isaacson, M. Khalkhali, S. Randriambololona.

COURSE COORDINATOR:  R.N. Bryan

TEXTBOOK:  


Midterm Tests and Final Exams for Calculus 1000A, published for the Department of Mathematics by Custom Course Materials.  (OPTIONAL)

PREREQUISITES:  One or more of Ontario Secondary School MCV4U, Mathematics 0110A/B, or the former Ontario Secondary School MCB4U.

ANTIREQUISITES:  Calculus 1100A/B, Applied Mathematics 1413, the former Mathematics 030.

COURSE OUTLINE:  Selected topics from Chapters 1-6.  See the List of Suggested Exercises for more details.

WHAT IS EXPECTED OF THE STUDENT?  
Students are responsible for the material presented in lectures and for learning how to solve the suggested exercises.

EVALUATION OF STUDENT PERFORMANCE:  
Midterm Examination:  (35%)  Friday, October 23, 2009, 7:00 p.m. - 9:30 p.m.
Final Examination:  (50%)  To be scheduled by the Registrar’s Office.  (3 hours).
Quizzes/Assignments:  (15%)

NOTE:  Quizzes and/or assignments will be described by your instructor.  The marks for individual sections will be normalized to the quiz marks for the section with the highest average pro-rated according to performance on the final exam.
POLICY ON CALCULATORS AND OTHER ELECTRONIC DEVICES:
Although the use of calculators and other electronic devices will not be permitted for the midterm test and the final examination, students are expected to have a reasonable facility in the use of their calculators.

IMPORTANT SENATE POLICY:
Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded and that all course prerequisites have been successfully completed. If the student does not have the requisites for a course, and does not have written special permission from his or her Dean to enroll in the course, the student may be removed from the course and it will be deleted from the student’s record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites.

STATEMENT ON ACADEMIC OFFENCES:
Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site:
Computer-marked multiple-choice tests and/or exams may be subject to submission for similarity review by software that will check for unusual coincidences in answer patterns that may indicate cheating.

MEDICAL EXCUSE REGULATIONS:
If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's Office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with his or her instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately.
For further information please see:

A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services.
The form can be found here:
https://studentservices.uwo.ca/secure/medical_document.pdf

WEB PAGE ADDRESS:  http://www.math.uwo.ca/~rmbryan/09calc1000Aindex.html
A1. Evaluate \( \cos \left[ 2 \tan^{-1} (2) \right] \).

\[ \begin{array}{c|c|c|c|c}
\text{A:} & \frac{3}{5} & \text{B:} & \frac{3}{5} & \text{C:} \\
\text{D:} & \frac{4}{5} & \text{E:} & -\frac{5}{3} \\
\end{array} \]

A2. Determine \( \lim_{x \to \infty} \arccos \left( \frac{1}{3} - x \right) \).

\[ \begin{array}{c|c|c|c|c}
\text{A:} & -\frac{\pi}{2} & \text{B:} & \frac{\pi}{2} & \text{C:} \\
\text{D:} & \pi & \text{E:} & -\pi \\
\end{array} \]

A3. If \( y = \sin^2 (3x) \), find \( \frac{dy}{dx} \).

\[ \begin{array}{c|c|c|c|c}
\text{A:} & 2 \sin (3x) \cos (3x) 3x \ln 3 & \text{B:} & 2 \sin (3x) \cos (3x) 3x & \text{C:} \\
\text{D:} & -2 \sin (3x) \cos (3x) 3x & \text{E:} & -2 \sin (3x) \cos (3x) 3x \ln 3 \\
\end{array} \]

A4. Determine \( \lim_{x \to 0^+} \frac{5 \sin x - 1}{x} \).

\[ \begin{array}{c|c|c|c|c}
\text{A:} & \ln 5 & \text{B:} & 1 & \text{C:} \\
\text{D:} & 5 & \text{E:} & \text{does not exist} \\
\end{array} \]

A5. Determine \( \lim_{x \to 1} \frac{x - 1}{\ln x - \sin (\pi x)} \).

\[ \begin{array}{c|c|c|c|c}
\text{A:} & \frac{1}{1 - \pi} & \text{B:} & 1 & \text{C:} \\
\text{D:} & \frac{1}{1 + \pi} & \text{E:} & \text{does not exist} \\
\end{array} \]

A6. Determine \( \lim_{x \to 0} \frac{\sin^{-1} (5x)}{x} \).

\[ \begin{array}{c|c|c|c|c}
\text{A:} & \frac{1}{5} & \text{B:} & 1 & \text{C:} \\
\text{D:} & 5 & \text{E:} & \text{does not exist} \\
\end{array} \]

A7. If \( y = 2^{\ln x} \), find \( \frac{dy}{dx} \).

\[ \begin{array}{c|c|c|c|c}
\text{A:} & 2^{1/x} & \text{B:} & 2^{1/x} \ln 2 & \text{C:} \\
\text{D:} & 2^{\ln x} \ln 2 & \text{E:} & \frac{2^{\ln x} \ln 2}{x} \\
\end{array} \]

A8. If \( f(x) = \ln (\arctan x) \), find \( f'(x) \).

\[ \begin{array}{c|c|c|c|c}
\text{A:} & \frac{1}{\arctan x} & \text{B:} & \frac{1}{\sqrt{1 - x^2} \arctan x} & \text{C:} \\
\text{D:} & \frac{1}{(1 + x^2) \sec^2 x} & \text{E:} & \frac{1}{(x^2 + 1) \arctan x} \\
\end{array} \]

A9. If \( x \ln y + y \ln x = 0 \), find \( \frac{dy}{dx} \) at the point \((x, y) = (1, 1)\).
A10. [2 marks] Which of the following integrals equals \( \lim_{n \to \infty} \sum_{i=1}^{n} \sin \left( \frac{\pi i}{n} \right)^2 \)?

<table>
<thead>
<tr>
<th>Option</th>
<th>Integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( \int_{0}^{\pi} \sin^2 x , dx )</td>
</tr>
<tr>
<td>B</td>
<td>( \int_{0}^{1} \sin^2 (\pi x) , dx )</td>
</tr>
<tr>
<td>C</td>
<td>( \int_{0}^{\pi} \sin (x^2) , dx )</td>
</tr>
<tr>
<td>D</td>
<td>( \int_{0}^{1} \sin \left( (\pi x)^2 \right) , dx )</td>
</tr>
<tr>
<td>E</td>
<td>( \int_{0}^{\pi} \sin (x^2) , dx )</td>
</tr>
</tbody>
</table>

A11. [2 marks] If \( f(x) = \int_{1}^{\pi} \arcsin t \, dt \), find \( f' \left( \frac{1}{2} \right) \).

<table>
<thead>
<tr>
<th>Option</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( \frac{5\pi}{6} )</td>
</tr>
<tr>
<td>B</td>
<td>( \frac{\pi}{2} )</td>
</tr>
<tr>
<td>C</td>
<td>( \frac{\pi}{3} )</td>
</tr>
<tr>
<td>D</td>
<td>( \frac{\pi}{6} )</td>
</tr>
<tr>
<td>E</td>
<td>( \frac{2\pi}{3} )</td>
</tr>
</tbody>
</table>

A12. [2 marks] Evaluate \( \frac{d}{dx} \int_{0}^{1} e^{\arctan x} \, dx \).

<table>
<thead>
<tr>
<th>Option</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( e^{\frac{\pi}{4}} - 1 )</td>
</tr>
<tr>
<td>B</td>
<td>( e^{\frac{\pi}{4}} )</td>
</tr>
<tr>
<td>C</td>
<td>( e^{\frac{\pi}{4}} )</td>
</tr>
<tr>
<td>D</td>
<td>( e^{\frac{\pi}{4}} - 1 )</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
</tr>
</tbody>
</table>

A13. [2 marks] If \( \int_{-2}^{-2} 3 f(x) \, dx = 12 \) and \( \int_{-2}^{5} f(x) \, dx = 6 \), find \( \int_{-2}^{5} f(x) \, dx \).

<table>
<thead>
<tr>
<th>Option</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>18</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>-4</td>
</tr>
<tr>
<td>E</td>
<td>-6</td>
</tr>
</tbody>
</table>

A14. [2 marks] Suppose \( \sum_{k=1}^{10} a_k = -2 \) and \( \sum_{k=1}^{10} b_k = 25 \). Find \( \sum_{k=1}^{10} (a_k + b_k - 1) \).

<table>
<thead>
<tr>
<th>Option</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
</tr>
<tr>
<td>B</td>
<td>13</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
</tr>
<tr>
<td>D</td>
<td>22</td>
</tr>
<tr>
<td>E</td>
<td>23</td>
</tr>
</tbody>
</table>

A15. [2 marks] If \( y = x^{\sqrt{x}} \) (for \( x > 0 \)), find \( \frac{dy}{dx} \).

<table>
<thead>
<tr>
<th>Option</th>
<th>Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( x^{\sqrt{x}} \left( \frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}} \right) )</td>
</tr>
<tr>
<td>B</td>
<td>( \frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}} )</td>
</tr>
<tr>
<td>C</td>
<td>( \sqrt{x} \left( x^{\sqrt{x}-1} \right) )</td>
</tr>
<tr>
<td>D</td>
<td>( \frac{x^{\sqrt{x}} \ln x}{2\sqrt{x}} )</td>
</tr>
<tr>
<td>E</td>
<td>( \frac{\sqrt{x} \left( x^{\sqrt{x}-1} \right)}{2} )</td>
</tr>
</tbody>
</table>

A16. [2 marks] If \( y = \ln (1 - x) \), find \( \frac{d^2 y}{dx^2} \) at \( x = \frac{1}{2} \).

<table>
<thead>
<tr>
<th>Option</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>-2</td>
</tr>
<tr>
<td>D</td>
<td>-4</td>
</tr>
<tr>
<td>E</td>
<td>-\ln 2</td>
</tr>
</tbody>
</table>
A17. [2 marks] If \( f'(x) = \frac{2}{1 + x^2} + \sec x \tan x \) and \( f(0) = 0 \), find \( f(x) \).

A: 2 \( \tan^{-1} x + \tan x \)  
B: 2 \( \sin^{-1} x + \sec x - 1 \)  
C: 2 \( \tan^{-1} x + \sec x \)  
D: 2 \( \sin^{-1} x + \tan x \)  
E: 2 \( \tan^{-1} x + \sec x - 1 \)

A18. [2 marks] Evaluate \( \int_0^1 \frac{1}{x (\ln x)^{1/3}} \, dx \) for \( x > 0 \).

A: \( \frac{3}{2} (\ln x)^{2/3} + C \)  
B: \( \frac{2}{3} (\ln x)^{2/3} + C \)  
C: \( (\ln x)^{4/3} + C \)  
D: \( \frac{3}{x} (\ln x)^{2/3} + C \)  
E: \( \frac{1}{x^{4/3}} + C \)

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

A19. [2 marks] Evaluate \( \int_0^1 \frac{2^x}{\ln 2} \, dx \).

A: \( \frac{1}{\ln 4} \)  
B: \( \frac{1}{(\ln 2)^2} \)  
C: 1  
D: \( \frac{1}{\ln 2} \)  
E: 1

A20. [2 marks] Choose the substitution which transforms \( \int \tan^2 x \sec^2 x \, dx \) into \( \int u^2 \, du \).

A: \( u = \sec x \)  
B: \( u = \sec x \tan x \)  
C: \( u = \tan x \)  
D: \( u = \tan^2 x \)  
E: \( u = \sec^2 x \)

A21. [2 marks] Evaluate \( \int_{-3}^3 \frac{1}{\sqrt{9 - x^2}} \, dx \) for \(-3 < x < 3\).

A: \( \sin^{-1} \left( \frac{x}{3} \right) + C \)  
B: \( \tan^{-1} \left( \frac{x}{3} \right) + C \)  
C: \( \frac{1}{3} \sin^{-1} \left( \frac{x}{3} \right) + C \)  
D: \( \frac{1}{3} \tan^{-1} \left( \frac{x}{3} \right) + C \)  
E: \( 2 (9 - x^2)^{1/2} + C \)

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

A22. [2 marks] Evaluate \( \int_{-2}^2 \sqrt{4 - x^2} \, dx \) by interpreting it as the area of a region.

A: \( \frac{\pi}{2} \)  
B: \( \pi \)  
C: \( 2\pi \)  
D: 4  
E: 0

A23. [2 marks] Let \( R \) be the shaded region, shown below, bounded by \( y = 2 - x^2 \) and \( y = x \). Choose the integral, the value of which is the area of \( R \).

A: \( \int_{-1}^2 (2 - x^2 - x) \, dx \)  
B: \( \int_{-2}^1 (2 - x^2 - x) \, dx \)  
C: \( \int_{-2}^1 (x - 2 + x^2) \, dx \)  
D: \( \int_{-1}^2 (x - 2 + x^2) \, dx \)  
E: \( \int_{-2}^1 (2 - x^2) \, dx \)

NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.

In problems A24 and A25, \( R \) is the shaded region, shown below, bounded by \( y = \sqrt{x^2 + 1} \), \( y = 0 \), \( x = 0 \) and \( x = \sqrt{3} \).
A24. [2 marks] Choose the integral which gives the volume of the solid generated by rotating $R$, shown above, about the $x$-axis.

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A25. [2 marks] Choose the integral which gives the volume of the solid generated by rotating $R$, shown above, about the $y$-axis.

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**PART B (50 marks)**

**NOTE: SHOW ALL YOUR WORK FOR PROBLEMS ON THIS PAGE.**

B26. [4 marks] Evaluate $\int_{0}^{3\pi/2} |\cos x| \, dx$.

B27. [5 marks] Determine $\lim_{x \to e^+} (\ln x)^{1/e}$.

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

B28. [5 marks] Find the area of the shaded region $R$, shown below, bounded by $x = 3 - y^2$ and $x = -\frac{y^2}{4}$.

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

B29. [5 marks] Evaluate $\int_{1}^{\sqrt{3}} \frac{(\arctan x)^2}{1 + x^2} \, dx$.

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

B30. [4 marks] Find $F'(x)$ if $F(x) = \int_{\sqrt{x}}^{2\sqrt{x}} \sin(t^2) \, dt$.

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

B31. [6 marks] Let $R$ be the shaded region, shown below, bounded by $x = y^2$ and $x = 1$. Find the volume of the solid generated by rotating $R$ about the line $y = 1$.

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

B32. [5 marks] Let $S$ be the solid with base $R$, the shaded region shown below, bounded by $y = 2\sin x$, $y = 0$ for $0 \leq x \leq \pi$. The cross-sections of $S$ perpendicular to the $x$-axis are squares. Find the volume of $S$.

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

B33. [5 marks] If $x \sin (\pi x) = \int_{0}^{x^2} f(t) \, dt$, where $f$ is continuous on $\mathbb{R}$, find $f'(4)$.

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

B34. [5 marks] The area of the region labeled $B$ is three times the area of the region labeled $A$ in the figure below. Express $b$ in terms of $a$.

B35. [6 marks] Evaluate $\int_{0}^{\pi/4} \frac{\sec^2 x}{(1 + 7 \tan x)^{2/3}} \, dx$. 
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<th>Subject</th>
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**INSTRUCTOR & COURSE EVALUATION**
THE UNIVERSITY OF WESTERN ONTARIO
### INSTRUCTOR & COURSE EVALUATION
#### THE UNIVERSITY OF WESTERN ONTARIO

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### Percent of Classes Attended
- 90% or more: 74
- 70% to 89%: 25
- 50% to 69%: 0
- 20% to 49%: 1

### Expected Grade
- A: 63
- B: 34
- C: 8
- D: 2

### Course Status
- Required: 74
- Optional: 30
- Total: 104

### Initial Level of Enthusiasm
- High: 37
- Medium: 54
- Low: 17
- Total: 108

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### Summary Statistics

- **Percent of Classes Attended**
  - 90% or more: 76
  - 70% to 89%: 22
  - 50% to 69%: 3
  - 20% to 49%: 1

- **Expected Grade**
  - A: 28
  - B: 34
  - C: 12
  - D: 0

- **Course Status**
  - Required: 58
  - Optional: 14
  - Total: 72

- **Initial Level of Enthusiasm**
  - High: 18
  - Medium: 33
  - Low: 25
  - Total: 76

- **Average of 1 to 14**
  - Mean: 5.42
  - Standard Deviation: 1.35
  - Median: 5
1.4.2 Calculus 1301

**Calculus 1301A/B - Calculus II**

For students requiring the equivalent of a full course in calculus at a less rigorous level than *Calculus 1501A/B*. Integration by parts, partial fractions, integral tables, geometric series, harmonic series, Taylor series with applications, arc length of parametric and polar curves, first order linear and separable differential equations with applications.

**Antirequisite(s):** *Calculus 1501A/B*, *Applied Mathematics 1413*.

**Prerequisite(s):** A minimum mark of 55% in one of *Calculus 1000A/B* or *1100A/B*.

**Corequisite(s):**

**Pre-or Corequisite(s):**

**Extra Information:** 4 lecture hours, 0.5 course.
INSTRUCTORS:
Dept. of Mathematics: J. Adamus, R. Bryan (King's), E. Coskun, S. Ditor,

TEXTBOOK: This is the same textbook as was used for Calculus 1000A and Calculus 1100A last term.

OTHER SUGGESTED MATERIALS
• Midterm Tests and Final Exams for Calculus 1301b, 2002-2003 through to 2008-09, published for the Department of Mathematics by Custom Course Materials. (OPTIONAL)
• The Mathematics Survival Kit, Revised Edition by Jack Weiner, published by Thomson/Nelson. (OPTIONAL)

PREREQUISITE: A minimum mark of 55% in one of Calculus 1000A/B or 1100A/B.

ANTIREQUISITES: Calculus 1501A/B, Applied Mathematics 1413.

SENATE POLICY on prerequisites: Unless you have either the requisites for this course or written special permission from your Dean to enroll in it, you will be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisites.

COURSE OUTLINE: This course continues from Calculus 1000A/B and 1100A/B. We will cover selected topics from Chapters 7-11. See the List of Suggested Exercises for more details.

WHAT IS EXPECTED OF THE STUDENT?
Students are responsible for learning the material presented in lectures, for learning how to solve the suggested exercises, and for demonstrating that learning on exams, quizzes, and/or assignments.

EVALUATION OF STUDENT PERFORMANCE:
• Midterm Examination: (35%) Friday, February 26, 2010, 7:00 - 9:30 pm
• Final Examination: (50%) To be scheduled by the Registrar's Office. (3 hours).
• Quizzes/Assignments: (15%)

NOTE: Quizzes and/or assignments will be described by your instructor. The marks in individual sections will be rescaled according to class performance for that section on the final exam in order to ensure that, overall, no section is advantaged (or disadvantaged) by having easier (or harder) quizzes or assignments.

HELP CENTRES: The Calculus 1301B Help Centres are scheduled as follows:
Mondays and Wednesdays: 4:30 - 6:30 p.m. in MC 106
Tuesdays and Thursdays: 4:30 - 6:30 p.m. in MC 204.
STATEMENT ON ACADEMIC OFFENCES:

Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site:


Computer-marked multiple-choice tests and/or exams may be subject to submission for similarity review by software that will check for unusual coincidences in answer patterns that may indicate cheating.

NEW MEDICAL EXCUSE REGULATIONS:

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's Office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately.

For further information please see:

A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services.

The form can be found here:
https://studentservices.uwo.ca/secure/medical_document.pdf

WEB PAGE:
http://apmaths.uwo.ca/~xzou/CAL1300b/main/html
Final

Multiple choice.

1. Evaluate $\int \cos^{-1} x \, dx$.
   (A) $x \cos^{-1} x + \sqrt{1 - x^2} + C$
   (B) $x \cos^{-1} x - \sqrt{1 - x^2} + C$
   (C) $- \sin^{-1} x + C$
   (D) $\ln |\sec x + \tan x| + C$
   (E) $1/\sqrt{1 - x^2} + C$

2. Evaluate $\int_1^{\infty} \frac{dx}{x^5}$ if the integral converges.
   (A) 1
   (B) $-1/4$
   (C) 1/4
   (D) integral diverges
   (E) 0

3. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n(x-4)^n}{5^n}$?
   (A) 5
   (B) 1/5
   (C) 4/5
   (D) 0
   (E) $\infty$

4. Determine if the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$ converges absolutely, converges conditionally, or diverges.
   (A) converges absolutely
   (B) converges conditionally
   (C) diverges

5. Find the function that is represented by the power series $\sum_{n=1}^{\infty} nx^n$ on $(-1, 1)$.
   (A) $\frac{1}{(1-x)^2}$
   (B) $\frac{x}{1-x}$
   (C) $\frac{1}{1-x}$
   (D) $-\frac{x}{(1-x)^2}$
   (E) $\frac{x}{(1-x)^2}$
6. Find the first non-zero terms of the power series representation of \( e^x \cos x \).
   (A) \( 1 + x + x^2 + x^3/6 + \ldots \)
   (B) \( 1 + x^2 - x^3/6 + \ldots \)
   (C) \( 1 + x - x^3/3 + \ldots \)
   (D) \( x + x^2 - x^3/6 + \ldots \)
   (E) \( 1 + 2x - x^3/3 + \ldots \)

7. Find the Maclaurin series for \( f(x) = e^{-x^2} \).
   (a) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!} \)
   (b) \( \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \)
   (c) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \)
   (d) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \)
   (e) \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n} \)

8. \( 1 - e + \frac{e^2}{2!} - \frac{e^3}{3!} + \frac{e^4}{4!} - \cdots \) equals
   (A) \( e^c \)
   (B) \( e^{-c} \)
   (C) \( e \)
   (D) \( e^{-1} \)
   (D) \( 1 \)

9. The differential equation \( y'(x) = \ln(2e^{3y}) \) is
   (A) Linear and separable
   (B) Separable but not linear
   (C) Linear but not separable
   (D) Neither linear nor separable
   (E) Not solvable

10. The length of one loop of the polar curve \( r = 2\sin \theta \) is equal to
    (a) \( 4\pi \)
    (b) \( \pi \)
    (c) \( 2\pi \)
    (d) \( 8\pi \)
11. Find the area of the region bounded by the polar axis and the polar curve \( r = \sqrt{\theta}, \ 0 \leq \theta \leq 2\pi \).

(a) \(2\pi^2\)
(b) \(\sqrt{2}\)
(c) \((2\pi)^{3/2}\)
(d) \(\sqrt{2\pi}\)
(e) \(\pi^2\)

12. Find a formula for the length of the curve given by \(y = \sqrt{1 - 4x^2}\).

(A) \(\int_{-1/2}^{1/2} \frac{4x}{\sqrt{1-4x^2}} dx\)
(B) \(\int_{-1/2}^{1/2} \sqrt{1 + (1 - 4x^2)^2} dx\)
(C) \(\int_{-1/2}^{1/2} \sqrt{1 - 4x^2} dx\)
(D) \(\int_{-1/2}^{1/2} \sqrt{\frac{1+12x^2}{1-4x^2}} dx\)
(E) \(\int_{-1/2}^{1/2} \frac{1}{\sqrt{1-4x^2}} dx\)

13. Determine the slope of the tangent line to the curve given by the equations \(x = 2 + \cos t, \ y = 1 + \sin t\) at the point \((2, 2)\).

(A) 2
(B) 3
(C) \(\sqrt{2}\)
(D) 0
(E) –2

14. Find the slope of the straight line that passes through the point \(A\) with polar coordinates \(r = 1, \ \theta = 0\), and the point \(B\) with polar coordinates \(r = 2, \ \theta = \pi/4\).

(A) \(\frac{\sqrt{2}}{\sqrt{2}+1}\)
(B) \(1 + \frac{1}{\sqrt{2}}\)
(C) \(1 - \frac{1}{\sqrt{2}}\)
(D) \(\frac{\sqrt{2}}{\sqrt{2}-1}\)
(E) 1
15. An integrating factor for the linear differential equation \( \frac{dy}{dx} + \frac{1}{x}y = \sin x \) is
   (A) \( x \)
   (B) \( e^x \)
   (C) \( \ln x \)
   (D) \( x^2 \)
   (E) \( \sin x \)

16. The curve whose polar equation is \( r = 2 \sec \theta \) has the Cartesian equation
   (A) \( y = 2 \)
   (B) \( x + y = 2 \)
   (C) \( x = 2 \)
   (D) \( x^2 + y^2 = 4x \)
   (E) \( x^2 + y^2 = 4y \)

17. The curve in the figure has the polar equation
   \begin{center}
   \includegraphics[width=0.3\textwidth]{circle.png}
   \end{center}
   (A) \( r = \cos \theta \)
   (B) \( r = 1 - \cos \theta \)
   (C) \( r = \sin \theta \)
   (D) \( r = (1 - \cos \theta)^2 \)
   (E) \( r = \frac{2\theta}{\pi} \)

18. A Cartesian equation of the tangent line to the polar curve \( r = 1 + 2\sin \theta \) at the point where \( \theta = \pi \) is
   (A) \( y = x + 1 \)
   (B) \( y = -1/2(x + 1) \)
   (C) \( y = -x - 1 \)
   (D) \( x = -1 \)
   (E) \( y = -2(x + 1) \)
19. The curve given by the parametric equations \( x = 3 \cos(\pi t) \), \( y = 3 \sin(\pi t) \) for \( 0.5 \leq t < 1.5 \) is
(A) half a circle
(B) the graph of a function \( y = h(x) \)
(C) a full circle
(D) a parabola
(E) does not exist

20. Find the solution of the differential equation \( \frac{dy}{dx} = x^3 y \) satisfying \( y(1) = 0 \).
(A) \( y = \frac{1}{4}x^4 - 1/4 \)
(B) \( y^2 = \frac{1}{2}x^4 - 1/2 \)
(C) \( \ln y = \frac{1}{4}x^4 \)
(D) \( y = e^{x^4-1} - 1 \)
(E) \( y = \ln |\frac{1}{2}x^4 + 1/2| \)

Non-multiple choice

1. Find the first three non-zero terms of the Taylor series of the function \( f(x) = \tan^{-1} x \) centred at \( x = 1 \).

2. Calculate the arc length of the curve given by the parametric equation \( x(t) = \cos(t) \), \( y(t) = t + \sin(t) \), \( 0 \leq t \leq \pi \).

3. Solve the initial value problem \( y' = xe^{-\sin x} - y \cos x \), \( y(0) = 1 \).

4. (i) Find the values of \( \theta \) for which the limaçon \( r = 1 - 2 \sin \theta \) passes through the origin.
(ii) Calculate the area of the inner loop of \( r = 1 - 2 \sin \theta \).

5. (i) Find the first three non-zero terms of the Maclaurin series for \( (1 - x^2)^{1/2} \)
(ii) Use (i) to evaluate \( \lim_{x \to 0} \frac{1 - x^2 - 1}{\cos x - 1} \).
### Instructor & Course Evaluation

**The University of Western Ontario**

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#### Calculations

- **Percent of Classes Attended**
  - 90% or more: 11
  - 70% to 89%: 6
  - 50% to 69%: 1
  - 20% to 49%: 0

- **Expected Grade**
  - A: 18
  - B: 9
  - C: 1
  - D: 0

- **Course Status**
  - Required: 22
  - Optional: 4
  - Total: 26

- **Initial Level of Enthusiasm**
  - High: 7
  - Medium: 14
  - Low: 7
  - Total: 28

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**INSTRUCTOR & COURSE EVALUATION**

**The University of Western Ontario**

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- **Percent of Classes Attended**
  - 90% or more: 11
  - 70% to 89%: 6
  - 50% to 69%: 1
  - 20% to 49%: 0

- **Expected Grade**
  - A: 18
  - B: 9
  - C: 1
  - D: 0

- **Course Status**
  - Required: 22
  - Optional: 4
  - Total: 26

- **Initial Level of Enthusiasm**
  - High: 7
  - Medium: 14
  - Low: 7
  - Total: 28

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## INSTRUCTOR & COURSE EVALUATION

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### Percent of Classes Attended

- 90% or more: 52
- 70% to 89%: 22
- 50% to 69%: 5
- 20% to 49%: 1

### Expected Grade

- A: 45
- B: 25
- C: 10
- D: 2

### Course Status

- Required: 60
- Optional: 20
- Total: 80

### Initial Level of Enthusiasm

- High: 20
- Medium: 47
- Low: 14

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## INSTRUCTOR & COURSE EVALUATION

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### Instructor & Course Evaluation

**The University of Western Ontario**

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1.4.3 Calculus 1501

**Calculus 1501A/B - Calculus II for Mathematical and Physical Sciences**

Students who intend to pursue a degree in Actuarial Science, Applied Mathematics, Astronomy, Mathematics, Physics, or Statistics should take this course. Techniques of integration; The Mean Value Theorem and its consequences; series, Taylor series with applications; parametric and polar curves with applications; first order linear and separable differential equations with applications.

**Antirequisite(s):** Calculus 1301A/B, Applied Mathematics 1413.

**Prerequisite(s):** A minimum mark of 60% in one of Calculus 1000A/B or 1100A/B.

**Corequisite(s):**

**Extra Information:** 4 lecture hours, 0.5 course.
INSTRUCTORS: Department of Applied Mathematics: A. Metzler  
Department of Mathematics: R. Bryan, R. Shafikov.

COURSE COORDINATOR: R.N. Bryan

TEXTBOOK: This is the same textbook as was used for Calculus 1000A and Calculus 1100A last term. **STUDENT VALUE PACKAGE: Single Variable Calculus, Sixth Edition, with Early Transcendentals, with Student Solutions Manual, Vol. 1** by James Stewart, and **Interactive Video Skillbuilder CD-ROM and Enhanced WebAssign Access Code**, published by Thomson/Brooks/Cole. (Shrink-wrapped package)


*Midterm Tests and Final Exams for Calculus 1501B* published for the Department of Mathematics by Custom Course Materials. *(OPTIONAL)*

PREREQUISITES: A minimum mark of at least 60% in one of Calculus 1000A/B or 1100A/B.

ANTIREQUISITES: Calculus 1301A/B, Applied Mathematics 1413.

COURSE OUTLINE: Selected topics from Chapters 4, 7, 8, 9, 10 and 11. See the list of Suggested Exercises for more details.

WHAT IS EXPECTED OF THE STUDENT? Unless you have either the requisites for this course or written special permission from your Dean to enroll in it, you may be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisites.

EVALUATION OF STUDENT PERFORMANCE:  
**First Midterm Examination:** (20%) Friday, February 5, 2010, 7:00 - 9:30 p.m.  
**Second Midterm Examination:** (25%) Friday, March 12, 2010, 7:00 - 9:30 p.m.  
**Final Examination:** (40%) To be scheduled by the Registrar's Office. (3 hours).  
**Quizzes/Assignments:** (15%)  
*NOTE:* Quizzes and/or assignments will be described by your instructor. The marks for individual sections will be normalized to the quiz marks for the section with the highest average pro-rated according to performance on the final exam.

CALCULATOR POLICY: Although the use of calculators will not be permitted for the midterm tests and the final examination, students are expected to have a reasonable facility in the use of their calculators.
STATEMENT ON ACADEMIC OFFENCES:
Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site:

NEW MEDICAL EXCUSE REGULATIONS:
If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's Office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately.
For further information please see:

A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services.
The form can be found here:
https://studentservices.uwo.ca/secure/medical_document.pdf

http://www.math.uwo.ca/~shafikov/1501  Professor Shafikov's section
1. [6 marks] Evaluate \[ \int \ln \left(1 + \frac{1}{x^2}\right) \, dx. \]

**Hint:** Use integration by parts.

2. [8 marks] Determine whether the improper integral \[ \int_1^\infty \frac{1}{x(x+1)^2} \, dx \]
converges or diverges. If it converges, find its value.

3. [8 marks] Find the radius of convergence and interval of convergence of the power series
\[ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{5^n \sqrt{n}}. \]

4. (a) [6 marks] Write the Maclaurin series for \( \sin(x^3) \).
   (b) Write the Maclaurin series for \( \ln(1-x^3) \).
   (c) Use series to evaluate \( \lim_{x \to 0} \frac{\sin(x^3) - x^3}{\ln(1-x^3) + x^3 + x^6/2} \).

5. (a) [10 marks] Approximate \( f(x) = \ln x \) by a Taylor polynomial of degree 3 (i.e., \( T_3(x) \)) with \( a = 1 \).
   (b) Using Taylor’s inequality, estimate the absolute value of the error by estimating the remainder corresponding to Part (a) for \( 1/2 \leq x \leq 3/2 \); i.e., estimate the accuracy of the approximation \( f(x) \approx T_3(x) \) for \( x \) in the interval \( [1/2, 3/2] \).

6. (a) [10 marks] State the Mean Value Theorem.
   (b) Use the Mean Value Theorem to show that if \( x < 0 \), then \( x < \tan^{-1} x \).

7. [6 marks] Solve the following initial value problem for \( y \). (An explicit solution is required.)
\[ \sqrt{1+x^2} \frac{dy}{dx} = \frac{1}{x^2} \quad \text{for} \quad x > 0; \quad y(1) = 2 \]

8. [8 marks] Solve the following initial value problem for \( y \). (An explicit solution is required).
\[ \frac{dy}{dx} + y = \frac{e^{-x}}{x(x^2 + 1)} \quad \text{for} \quad x > 0; \quad y(1) = 0 \]

9. [10 marks] A 400 litre, open-topped tank initially contains 100 litres of brine (salt water) in which 5 kilograms of salt are dissolved. Starting at time \( t = 0 \), brine containing .01 kilograms of salt per litre flows into the tank at the rate of 3 litres per minute and the well-stirred mixture simultaneously flows out of the tank at the rate of 1 litre per minute.
   (a) Give a formula for the number of kilograms of salt in the tank at any time \( t \) before the tank overflows.
   (b) How many kilograms of salt are in the tank at the instant it overflows?

10. [10 marks] Let \( C \) be the curve given parametrically by \( x = \ln(\sec t + \tan t) - \sin t; \ y = \cos t \) for \( 0 \leq t \leq \frac{\pi}{3} \).
   (a) Find \( \frac{dy}{dx} \) in terms of \( t \).
(b) Find an equation of the tangent line to $C$ at the point where $t = \frac{\pi}{6}$.

(c) Find the length of $C$ as $t$ goes from 0 to $\frac{\pi}{3}$.

11. [10 marks] Find the length of the polar curve given by $r = \cos^3\left(\frac{\theta}{3}\right)$ for $-\pi \leq \theta \leq \pi$.

12. [8 marks] Let $C$ be the polar curve given by $r = \sin 3\theta$.

(a) Sketch $C$.

(b) Find the area enclosed by the one loop (leaf, petal) of $C$. 
## Instructor & Course Evaluation

**The University of Western Ontario**

<table>
<thead>
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<th>Teaching Faculty</th>
<th>Subject</th>
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### Percent of Classes Attended
- 90% or more: 34
- 70% to 89%: 5
- 50% to 69%: 1
- 20% to 49%: 0

### Expected Grade
- A: 33
- B: 5
- C: 3
- D: 0

### Course Status
- Required: 27
- Optional: 12
- Total: 39

### Initial Level of Enthusiasm
- High: 24
- Medium: 13
- Low: 3
- Total: 40

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Calculus 2502A/B - Advanced Calculus I

Differential calculus of functions of several variables: level curves and surfaces; limits; continuity; partial derivatives; total differentials; Jacobian matrix; chain rule; implicit functions; inverse functions; curvilinear coordinates; derivatives; the Laplacian; Taylor Series; extrema; Lagrange multipliers; vector and scalar fields; divergence and curl.

Antirequisite(s): Calculus 2302A/B, or the former Applied Mathematics 290a.

Prerequisite(s): A minimum mark of 60% in Calculus 1501A/B or Applied Mathematics 1413, or Calculus 1301A/B with a mark of at least 85%.

Corequisite(s):

Pre-or Corequisite(s): Linear Algebra 1600A/B or the former Mathematics 202a.

Extra Information: 3 lecture hours, 0.5 course.
Calculus 2502a: Advanced Calculus I, Fall 2008

Please check this page regularly for updates and announcements.

The first class is on Friday, September 5, 2008, and will be a regular class.

<table>
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<tr>
<td><strong>Instructor:</strong></td>
<td>Nicole Lemire</td>
</tr>
<tr>
<td><strong>Office:</strong></td>
<td>Middlesex 103C</td>
</tr>
<tr>
<td><strong>Phone:</strong></td>
<td>519-661-2111 x86533</td>
</tr>
<tr>
<td><strong>E-mail:</strong></td>
<td>nlemire at uwo dot ca</td>
</tr>
<tr>
<td><strong>Office Hours:</strong></td>
<td>W 2:30-3:30pm, Th 2:30-3:30pm</td>
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<td><strong>TAs:</strong></td>
<td>Mehdi Garrousian</td>
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**Text:**


A minimum mark of 60% in Calculus 1501A/B (051a/b) or

**Prerequisites:**

Applied Math 1413 (026), or a minimum mark of 85% in Calculus 1301A/B (Calculus 081a/b). (Old course numbers in parentheses.)

**Pre- or Corequisites:**

*Linear Algebra 1600A/B (040a/b)* or the former Mathematics 202a

**Antirequisites:**

Calculus 2302A/B (280a/b) or the former Applied Mathematics 290a.

**Course outline:**

69
Differential calculus of functions of several variables: level curves and surfaces; limits; continuity; partial derivatives; total differentials; Jacobian matrix; chain rule; implicit functions; inverse functions; curvilinear coordinates; derivatives; the Laplacian; Taylor Series; extrema; Lagrange multipliers; vector and scalar fields; divergence and curl.

We will also cover the required background about vectors in 3-dimensional space, e.g. dot products, cross products, lines, planes, etc.

The course material corresponds to Chapters 12, 13 and 14 and Sections 16.1 and 16.5 from the text, plus a few additional topics.

Link to UWO course calendar.

**Evaluation of Student Performance:**

- **Assignments:** 15%
- Midterm Examination: 35%
- Final Examination: 50%

There will be 6 assignments with due dates: Friday, September 19; Friday, October 3; Friday, October 17; Monday, November 3; Monday, November 17; Friday, November 28. (Every two weeks.) They are due at the beginning of class. Click here for a list of exercises and the procedures for handing in homework. The lowest (or missing) assignment grade will be dropped when computing the course grade. Here is what the assignments are out of: HW1 30, HW2 60, HW3 60, HW4 TBA, HW5 TBA, HW6 TBA.

**Midterm Information:**

The midterm exam will be on Thursday, October 23, 7-10 pm.

The midterm exam will cover the material of the first 3 assignments. That is, Chapters 12 and 13 and cylindrical and spherical coordinates from 15.7 and 15.8 of the text.

Here are some old midterms for practice: 2004 and 2006.

Note that the old midterms cover some material in Chapter 14. Our midterm will not cover this material.

Warning: Don't assume that the midterm will be a variant of the posted midterms. These are just here for practice. Study notes, text, homework and recommended problems. It is ill-advised to just study the old midterms.

**Room Assignments for Midterm:**

- **Section 1:** Students with Last Names beginning with ADA-PEN inclusive will write in SSC 2024
- **Section 1:** Students with Last Names beginning with PER-ZHU inclusive will write in SSC 3010
Section 2: All students will write in SSC 3018

Final Exam Information

The final exam will be held on Monday, December 8, 2008 from 9am-12 noon.

We will not be posting an old exam. Study homework and recommended problems, the text, and class notes. The exam will cover the entire course, but will emphasize material not already covered on the midterm.

There will be two help centres for the final exam.

Help Centre I: Nicole Lemire; Thursday, December 4, 2008 from 1-2pm in MC 106.

Help Centre II: Tatyana Foth; Friday, December 5, 2008 from 1-2pm in MC 106.

Room Assignments for Final:

Section 1: Students with Last Names beginning with ADAM-SILV inclusive will write in HSB 240.

Section 1: Students with Last Names beginning with SONG-ZHAN inclusive will write in SSC 2032.

Section 2: All students will write in SSC 2032.

For both exams, questions will be similar to homework and recommended exercises. The best way to prepare is to do all of those exercises, plus as many additional questions as you can.

Please read the section below if you have a conflict with either exam.

There will be help sessions before both exams. Both exams will contain a mix of multiple choice questions and long answer questions. Calculators are not permitted at either exam.

Make-up exams and conflicts

If you know ahead of time that you are unable to attend a midterm or final exam, you must let your instructor know at least two weeks in advance so alternative arrangements can be made. See also the University's policy on final exam conflicts.

For the midterm:

In the event that you must miss the midterm due to a valid conflict, the make-up exam will be given before the regular exam. Usually this will be earlier on the same day, or on the day before. two weeks before the exam date.

For the final exam:

If you have a conflict with another final exam, you must contact Rob Downes in the Registrar's Office by mid November to arrange a special time/place to write the final. If you have a conflict with a midterm for a year-long course, you must contact the instructor of the other course. If you have three final exams in 3...
If you are unable to attend a midterm or final exam due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately. For further information please see this link and the Student Services web site.

A student requiring academic accommodation due to illness should bring a Student Medical Certificate with them when visiting an off-campus medical facility and use a Record Release Form for visits to Student Health Services. Hard copies of both of these forms are available from your home Faculty Academic Counselling Service.

Failure to follow these rules will result in a grade of zero.

See also the University's policy on final exam conflicts.

What is expected of the student:

The aim of the course is for you to learn the techniques of multivariable calculus and to gain an understanding of the concepts on which the techniques are based. To accomplish this, it will be necessary to attend all classes, do all of the suggested exercises, and keep up to date with the material. We highly recommend reading the text and doing the exercises throughout the term, not only immediately before tests and assignment due dates.

The assignments are to be thought of as take-home exams and should be done entirely on your own. All those involved in copying will receive a negative grade on the assignment in question. In addition, all academic offenses will be reported to the Dean's office, and may result in further penalties. Regarding Scholastic Offences and Penalties, see pp. 39-41 of the Academic Calendar or refer to Academic Penalties.

A note to all students from the office of the Dean of the Faculty of Science:

You are responsible for ensuring that you have successfully completed all course prerequisites and that you have not taken an antirequisite course. Lack of prerequisites may not be used as the basis of appeal. If you are not eligible for a course, you may be removed from it at any time, and will receive no adjustment to your fees. These decisions cannot be appealed.

If you do not have the course prerequisites, and have not been granted a special permission to take the course by the department, it is in your best interest to drop the course well before the end of the add period. Your prompt attention to this matter will not only help protect your record, but will ensure that spaces become available for students who require this course for graduation.

Please check the exercises page and the course web page regularly for announcements and other information.
PART A: MULTIPLE CHOICE

1. (5pts) Answer the following questions by circling the right answer. Bold letters represent vectors in $\mathbb{R}^3$.

   (a) **True or False**: $\text{curl(curl } \mathbf{F}) = \mathbf{0}$.

   (b) **True or False**: If $f(x, y)$ has continuous second order partial derivatives, with

   $$f_x(0, 0) = f_y(0, 0) = f_{xx}(0, 0) = 0 \text{ and } f_{xy}(0, 0) = 1,$$

   then the origin is a saddle point for $f(x, y)$.

   (c) **True or False**: Let $\mathbf{r}(s) = (x(s), y(s), z(s))$ be a smooth curve parametrized by the arclength. Suppose $\mathbf{r}(0) = (0, 0, 0)$ and $\mathbf{r}(10) = (0, -1, 2)$. Then the length of the curve $\mathbf{r}(s)$ between the origin and the point $(0, -1, 2)$ is equal to 10.

   (d) **True or False**: If $f(x, y)$ is a function such that partial derivatives $f_x$ and $f_y$ exist for all $(x, y)$ in a neighbourhood of a point $(a, b)$, then $f(x, y)$ is differentiable at $(a, b)$.

   (e) **True or False**: If a function $f(x, y)$ is differentiable at a point $(a, b)$, then $f(x, y)$ is continuous at $(a, b)$.

2. (3pts) Determine which vector field is represented on the figure.

   (a) $\mathbf{F}(x, y) = y \mathbf{i} - x \mathbf{j}$

   (b) $\mathbf{F}(x, y) = \cos y \mathbf{i} + \cos x \mathbf{j}$

   (c) $\mathbf{F}(x, y) = \sin y \mathbf{i} + \sin x \mathbf{j}$

   (d) $\mathbf{F}(x, y) = \frac{x}{x^2+y^2} \mathbf{i} + \frac{y}{x^2+y^2} \mathbf{j}$

   (e) $\mathbf{F}(x, y) = (x^3 - 3x) \mathbf{i} + (y^3 - 3y) \mathbf{j}$
3. (3pts) The contour map of a polynomial \( g(x, y) \) is given on the figure. Choose the best description of \( g(x, y) \).

(a) \( g(x, y) \) has 2 critical points, of which one is local maximum and one is local minimum
(b) \( g(x, y) \) has 2 critical points, one of which is a saddle point
(c) \( g(x, y) \) has 4 critical points, two of which are saddle points
(d) \( g(x, y) \) has 4 critical points, of which two are local minima and two are local maxima
(e) \( g(x, y) \) has 4 critical points, which are all saddle points

**PART B: SHOW ALL WORK**

4. (6pts) For the surface whose equations in spherical coordinates is

\[
\rho^2 (\sin^2 \phi - 4 \cos^2 \phi) = 1
\]

(i) Find an equation of this surface in Cartesian coordinates.

(ii) Identify the surface.

5. (7pts) Let \( C \) be the curve of intersection of the surfaces \( x^2 + y^2 - z^2 = 1 \) and \( y - z = 1 \). Find parametric equations for \( C \).

6. (7pts) Find all the points at which the function

\[
f(x, y) = \begin{cases} 
xy - y^3, & \text{if } (x, y) \neq (0, 0) \\
x^2 + y^2, & \text{if } (x, y) = (0, 0)
\end{cases}
\]

is continuous. Justify your answer.
7. (8pts) Find the indicated derivatives.
   (i) \( w = (x^y + z^x)^z \). Find \( \frac{\partial w}{\partial x} \).
   (ii) \( f(x, y) = \tan^{-1}(\frac{y}{x}) \). Find \( f_x \) and \( f_{xy} \).

8. (8pts) Let \( z = f(x, y) = \sqrt[3]{x^2 - y^2} \).
   (i) Find the differential \( dz \).
   (ii) Find an equation of the tangent plane to the graph of \( f \) at \( (3, 1, 2) \).

9. (8pts) Let \( f(x, y) = x^2 - y \).
   (i) Find the maximum rate of change of \( f \) at \( (1, 0) \) and the direction in which it occurs.
   (i) Find \( D_uf(1, 0) \), where \( u = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) \).

10. (8pts) Let \( f(x, y, z) = z \sin(x + y) \).
    (i) Find \( \nabla f \).
    (ii) Find all critical points of \( f \) (do not classify). Plot the critical points on the \( (x, y) \) coordinate plane.

11. (8 pts) Let the function \( z = f(x, y) \) be given implicitly by
    \[ xyz + xye^z = 1. \]
    (i) Find \( \frac{\partial f}{\partial x} \).
    (ii) Evaluate \( \frac{\partial f}{\partial x}(1, 1) \).

12. (7pts) For the function
    \[ f(x, y) = x^3 - 6xy + 8y^3 \]
    identify each critical point as a local maximum, minimum, or a saddle point.

13. (7pts) Determine whether the vector field
    \[ \mathbf{F}(x, y, z) = \ln(x + y)\mathbf{i} + \frac{1}{x}\mathbf{j} + z^2\mathbf{k} \]
    is conservative or not. Carefully explain your argument.

14. (7pts) Let \( f(x, y, z) \) and \( g(x, y, z) \) be functions that have continuous second order partial derivatives. Prove that
    \[ \text{div} (\nabla f \times \nabla g) = 0. \]
15. (8pts) Find the extreme values of the function

\[ f(x, y, z) = x + y + 2z \]

on the region described by the inequality \( x^2 + y^2 + z^2 \leq 1 \).
### Instructor & Course Evaluation

**The University of Western Ontario**

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### Course Status

- **Required**: 26
- **Optional**: 6
- **Total**: 32

### Initial Level of Enthusiasm

- **High**: 18
- **Medium**: 9
- **Low**: 5
- **Total**: 32

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### Instructor & Course Evaluation

**The University of Western Ontario**

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1.4.5 Linear Algebra 1600

**Linear Algebra 1600A/B - Linear Algebra I**

Properties and applications of vectors; matrix algebra; solving systems of linear equations; determinants; vector spaces; orthogonality; eigenvalues and eigenvectors.

**Antirequisite(s):** Applied Mathematics 1411A/B, 2811B.

**Prerequisite(s):** One or more of Ontario Secondary School MCV4U, the former Ontario Secondary School MGA4U, Mathematics 1229A/B, the former Mathematics 017a/b, Calculus 1100A/B, or Calculus 1000A/B taken as a pre- or co-requisite.

**Corequisite(s):**

**Pre-or Corequisite(s):**

**Extra Information:** 3 lecture hours, 1 laboratory hour, 0.5 course.
Linear Algebra 1600B (Winter 2009/2010)

Please check the exercises page and this course web page regularly for announcements and updates.

| Instructor: | Matthias Franz | Samuel Isaacson |
| Office: | MC 134 | MC 135 |
| Phone extension: | 86540 | 86527 |
| E-mail (both at uwo dot ca): | mfranz | sisaaco |
| Office Hours: | TBA | TBA |
| TA Help Centres: | CHANGE Mon 4:30-5:30, Tues 4:30-5:30, Fri 12-1 in MC107 (except Oct 12) Thu 10:30-11:30, MC17 Students from any section can go to any help centre. |
| Class times: | MWF 12:30-1:30 | MWF 11:30-12:30 |
| Class location: | B&GS 0153 | NCB 113 |
| Tutorials: | 1 hour per week. The TA reviews material from the course and answers questions, and the tutorials also include quizzes (see below). You must attend the tutorial you are registered for (see your schedule). |
| Course outline: | Properties and applications of vectors; matrix algebra; solving systems of linear equations; determinants; vector spaces; independence; orthogonality; eigenvalues and eigenvectors. Link to UWO course calendar. |
| Text: | Contemporary Linear Algebra, by Anton and Busby. Wiley. Student Solutions Manual For Contemporary Linear Algebra, by Anton and Busby. Wiley. (Recommended.) You can buy them separately, or get both together for a good price. Both are available at the bookstore. It should be easy to find used copies as well. There is also a list of errata for the text. |
| Prerequisites: | One or more of Ontario Secondary School MCV4U, the former Ontario Secondary School MGA4U, Mathematics 1229A/B, the former Mathematics 017a/b, Calculus 1100A/B, or Calculus 1000A/B (formerly 050a/b) taken as a pre- or co-requisite. |
| Antirequisites: | Applied Mathematics 1411A/B (formerly 025a/b), 2811B (formerly 213b). |
| Web page: | This page is available at http://www.math.uwo.ca/~mfranz/courses/2009_1600b/, where you should also check for course announcements. Webct will not be used much, and contains a link to this page. |
| Quizzes: | There will be 11 quizzes throughout the year, during the tutorials, during the weeks shown below. The questions on the quizzes will be based on the recommended homework questions and will cover the material up to and including what was covered on Monday's lecture. TODO: no quiz during midterm week |
| Jan 6-7: | no quizzes | Feb 24-25: quiz 6 |
| Jan 13-14: | quiz 1 | Mar 3-4: quiz ? |
| Jan 20-21: | quiz 2 | Mar 10-11: quiz 7 |
| Jan 27-28: | quiz 3 | Mar 17-18: quiz 8 |
| Feb 3-4: | quiz 4 | Mar 24-25: quiz 9 |
| Feb 10-11: | quiz 5 | Mar 31-Apr 1: quiz 10 |
| Feb 17-18: | no tutorials | Apr 7-8: quiz 11 |
| The tutorials do run Jan 6-7 and TODO, and the TA will use the full time for going over course material and answering questions. |
| Midterm exam: | TBA |
| Final exam: | TBA |
| Evaluation: | Quizzes: 20%, Midterm: 30%, Final exam: 50%. For the quizzes, the lowest two scores will be dropped. |

What is expected of the student

The aim of the course is for you to learn the techniques of linear algebra and to gain an understanding of the concepts on which the techniques are based. To accomplish this, it will be necessary to attend all classes and tutorials, do all of the suggested exercises, and keep up to date with the material.

You are expected to read the text ahead of time to prepare for each lecture. The instructor will assume you have looked over the material before the lecture. You should also do the recommended exercises as the material is being covered, and then do them again before quizzes and exams.

This course covers a lot of material, and is cumulative (much more than other courses!), so it will be necessary to work hard throughout the term in order to do well.
Quizzes and Exams

For quizzes and exams, questions will be similar to the recommended exercises. The best way to prepare is to do all of those exercises, plus as many additional questions as you can. You should also study the text and your lecture notes, so you understand the concepts behind the problems you are solving.

Missed quiz, midterm or final exam

Remember that the lowest two quiz grades are dropped, to take into account absences for unforeseen reasons.

If you know ahead of time that you are unable to attend a quiz, midterm or final exam, you must let your instructor know at least two weeks in advance so alternative arrangements can be made. For final exam conflicts, see below.

For absence due to flu-like symptoms, please follow the instructions given by the online reporting system.

If you are unable to attend a quiz, midterm or final exam due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean’s office as soon as possible and contact your instructor immediately. It is the student’s responsibility to make alternative arrangements with their instructor. For further information please see this link and the Student Services web site.

A student requiring academic accommodation due to illness should bring a Student Medical Certificate with them when visiting an off-campus medical facility and use a Record Release Form for visits to Student Health Services. Hard copies of both of these forms are available from your home Faculty Academic Counselling Service.

If a quiz is missed and sufficient documentation is provided, the grade for that quiz will be reweighted to the other quizzes. If an exam is missed and sufficient documentation is provided, a make-up exam will be offered.

Failure to follow these rules may result in a grade of zero.

Final exam conflicts

Please see the University's policy on final exam conflicts. Here are the first two paragraphs:

A student who is scheduled to write more than two examinations in any 23-hour period may request alternative arrangements through the office of the dean of their faculty.

A student who is scheduled to write two examinations concurrently must notify the Registrar so that arrangements may be made for both examinations to be written in the Examination Conflict Room in a sequence established by the Registrar.

Please also let your instructor know about the conflict, and read the entire University policy.

Academic Offences

Scholastic offences are taken seriously and students are directed to read the official policy. Note that the penalty for cheating can include receiving a grade of zero in the course and suspension or expulsion from the University.

A note to all students from the office of the Dean of the Faculty of Science

You are responsible for ensuring that you have successfully completed all course prerequisites and that you have not taken an antirequisite course. Lack of prerequisites may not be used as the basis of appeal. If you are not eligible for a course, you may be removed from it at any time, and will receive no adjustment to your fees. These decisions cannot be appealed.

If you do not have the course prerequisites, and have not been granted a special permission to take the course by the department, it is in your best interest to drop the course well before the end of the add period. Your prompt attention to this matter will not only help protect your record, but will ensure that spaces become available for students who require this course for graduation.

Matthias Franz, 2010-01-03
1. For each of the following, circle T if the statement is always true and circle F if it can be false. 
Correct answer: 2 marks; no answer: 0 marks; incorrect answer: -1 mark for each part.
(a) If a matrix $B$ is obtained from $A$ by row operations, then $\text{null}(B) = \text{null}(A)$. T F
(b) If $A$ is a matrix with more rows than columns, then the equation $Ax = 0$ always has a nonzero solution. T F
(c) Every inhomogeneous linear system $Ax = b$ has either zero or one or infinitely many solutions. T F
(d) If $A$ is an invertible square matrix, then $\det(A^{-1}) = -\det(A)$. T F
(e) For any square matrix $A$, the characteristic polynomials of $A$ and $A^T$ are the same. T F
(f) Reflection about the $x$-axis in $\mathbb{R}^2$ is a one-to-one linear transformation. T F
(g) $\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ are similar. T F
(h) If $A$ is a $3 \times 3$ matrix with characteristic polynomial $(\lambda + 1)^3$, then $A$ is diagonalizable. T F

2. If $A$ is a $3 \times 5$ matrix with rank 1, compute the following. (No need to show work, and no part marks given.)
(a) $\dim(\text{row}(A)) = \underline{\phantom{0}}$
(b) $\dim(\text{col}(A)) = \underline{\phantom{0}}$
(c) $\dim(\text{null}(A)) = \underline{\phantom{0}}$
(d) $\dim(\text{null}(A^T)) = \underline{\phantom{0}}$

3. Let $A = \begin{bmatrix} 1 & 2 & 8 & 1 & -1 \\ 2 & -1 & 1 & 1 & 1 \\ 3 & 0 & 6 & 1 & -1 \\ 4 & -1 & 5 & 1 & -1 \end{bmatrix}$ and $R = \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.
Given that $R$ is the reduced row-echelon form of $A$, find the following:
(a) $\text{rank}(A)$
(b) A basis for $\text{row}(A)$
(c) A basis for $\text{col}(A)$
(2 pts)  (d) Let $T_A : \mathbb{R}^5 \rightarrow \mathbb{R}^4$ be the transformation associated to $A$. Is $T$ onto? Explain.

(3 pts)  (e) Find a basis for null($A$)

(5 pts)  4. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$. Compute $A^{-1}$.

(5 pts)  5. Do the vectors

$$v_1 = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 0 \\ -2 \\ -1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 2 \end{bmatrix}$$

form a basis of $\mathbb{R}^4$? Explain fully.

6. Let $A = \frac{1}{5} \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$.

(3 pts)  (a) Determine whether or not $A$ is an orthogonal matrix.

(3 pts)  (b) Let $T_A : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the transformation with standard matrix $A$. Does $T$ preserve lengths? Explain.

(5 pts)  7. Find the slope and intercept of the least squares line $y = a + bx$ of best fit for the four points $(0, 7), (1, 2), (1, 0), \text{ and } (2, -1)$. (Show all work!)

8. The vectors

$$w_1 = (1, 1, 1), \quad w_2 = (-1, 1, 3) \quad \text{and} \quad w_3 = (-2, 6, 2)$$

form a basis for $\mathbb{R}^3$.

(5 pts)  (a) Use the Gram-Schmidt process on the vectors above to find an orthogonal basis $\{v_1, v_2, v_3\}$ for $\mathbb{R}^3$ that contains a multiple of $w_1$.

(3 pts)  (b) Compute the coordinates of $x = (1, 2, -3)$ with respect to the basis $\{v_1, v_2, v_3\}$ you found in (a).

(3 pts)  (c) Use the previous parts to compute the orthogonal projection of $x$ onto $W = \text{span}\{w_1, w_2\}$.

9. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by $T(x, y) = (x - y, 2x - 2y)$.

(2 pts)  (a) Compute the standard matrix $[T]$ of $T$.

(2 pts)  (b) The vectors $v_1 = (1, 2)$ and $v_2 = (2, 3)$ form a basis $B$ for $\mathbb{R}^2$.

Compute $P_{B \rightarrow S}$, the change of basis matrix from $B$ coordinates to standard coordinates.

(2 pts)  (c) Compute $P_{S \rightarrow B}$, the change of basis matrix from standard coordinates to $B$ coordinates.

(Continued on next page.)

(2 pts)  (d) Compute $[T]_B$.

(2 pts)  (e) Compute a basis for $\text{ker}(T)$.

10. Let $A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$.

(2 pts)  (a) Compute the characteristic polynomial of $A$.

(2 pts)  (b) Find all eigenvalues of $A$ and their algebraic multiplicities.

(Continued on next page.)

(4 pts)  (c) Find bases for the eigenspaces of $A$.

(3 pts)  (d) Is $A$ diagonalizable? If so, give matrices $P$ and $D$ so that $P^{-1}AP = D$.

Did you remember to CHECK your answers where possible? Did you give full explanations and show all of your work?
### INSTRUCTOR & COURSE EVALUATION

**THE UNIVERSITY OF WESTERN ONTARIO**

<table>
<thead>
<tr>
<th>Instructor Name</th>
<th>Teaching Faculty</th>
<th>Teaching Department</th>
<th>Subject</th>
<th>Course Number</th>
<th>Section</th>
<th>Year Term</th>
<th>Enrolment</th>
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#### Question

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**THE UNIVERSITY OF WESTERN ONTARIO**

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84
## Instructor & Course Evaluation

**The University of Western Ontario**

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### 1.4.6 Mathematics 0110

**Mathematics 0110A/B - Introductory Calculus**

Introduction to differential calculus including limits, continuity, definition of derivative, rules for differentiation, implicit differentiation, velocity, acceleration, related rates, maxima and minima, exponential functions, logarithmic functions, differentiation of exponential and logarithmic functions, curve sketching.

**Antirequisite(s):** Mathematics 1225A/B, Calculus 1000A/B, Calculus 1100A/B, **Applied Mathematics 1413**, the former Mathematics 030.

**Prerequisite(s):** One or more of Ontario Secondary School MCF3M, MCR3U, or equivalent.

**Corequisite(s):**

**Pre-or Corequisite(s):**

**Extra Information:** 4 lecture hours, 0.5 course.
THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 0110A

2008-2009

INSTRUCTORS: S. Camiletti (King’s), C. Florence (Brescia), S. Joyner, S. Kuzmin (Huron), V. Okls.


PREREQUISITES: One or more of Ontario Secondary School MCF3M, MCR3U, or equivalent.


COURSE OUTLINE: Limits, continuity, definition of derivative, rules for differentiation, implicit differentiation, velocity, acceleration, related rates, maxima and minima, exponential function, logarithmic function, differentiation of exponential and logarithmic functions, curve sketching.

WHAT IS EXPECTED OF THE STUDENT:
regular attendance at classes; completion of all assigned work; assumption of complete responsibility for any classes, tests, or assignments which are missed; use of materials posted on the course WebCT web site.

EVALUATION OF STUDENT PERFORMANCE:
There will be in-class quizzes or other work counting for 10% of the grade. The two Term Tests will count for 50% (in total). There will be a three-hour Final Examination worth 40% of the grade. No make-up exams will be given except in the event of an authorized, documented emergency. A mark of 0 will be recorded for any test or examination that you have missed.

NOTE: If the student’s final exam mark is higher than a test mark, the weight from that test will be shifted to the final exam. (This may apply to either, or both, of the tests.)

EXAMINATION DATES:
The term tests will take place on
Friday, October 3, 2008, from 7:00-8:30 p.m.
and Friday, October 31, 2008, from 7:00-8:30 p.m.
The final examination will be scheduled by the Registrar during the midyear examination period.

NOTE: No calculators, notes or other aids are allowed on any quiz, test or exam in this course.
IMPORTANT SENATE POLICY:
Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded and that all course prerequisites have been successfully completed. If the student does not have the prerequisites for a course, and does not have written special permission from his or her Dean to enroll in the course, the student may be removed from the course and it will be deleted from the student’s record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites.

STATEMENT ON ACADEMIC OFFENCES:
Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site:
Computer-marked multiple-choice tests and/or exams may be subject to submission for similarity review by software that will check for unusual coincidences in answer patterns that may indicate cheating.

NEW MEDICAL EXCUSE REGULATIONS:
If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's Office as soon as possible and contact your instructor immediately. It is the student’s responsibility to make alternative arrangements with his or her instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately. For further information please see:

A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services. The form can be found here:
https://studentservices.uwo.ca/secure/medical_document.pdf
PART A - Multiple Choice (40 marks)

Circle your answer in each question below and mark it on the (scantron) answer sheet. Code as you go. Extra time will NOT be given for coding answers at the end of the exam. Be advised that ONLY THE SCANTRON CARD WILL BE MARKED IN THIS SECTION.

A1. [1 mark] What is the domain of the function \( f(x) = \sqrt{1-x} \)?
   \[ \begin{array}{|c|}
   \hline
   A: [1, \infty) & B: (-\infty, -1) \cup (1, \infty) & C: (-1, 1) & D: (-\infty, 1] & E: (-\infty, \infty) \\
   \hline
   \end{array} \]

A2. [1 mark] What is the domain of the function \( g(x) = \frac{1}{x^2 - 1} \)?
   \[ \begin{array}{|c|}
   \hline
   A: [1, \infty) & B: (-\infty, 1) \cup (1, \infty) & C: (-\infty, \infty) \\
   \hline
   D: (-\infty, -1) \cup (-1, 1) \cup (1, \infty) & E: \text{not defined} \\
   \hline
   \end{array} \]

A3. [1 mark] What is the range of the function \( f(x) = (2-x)^2 \)?
   \[ \begin{array}{|c|}
   \hline
   A: (-\infty, 2) \cup (2, \infty) & B: (-\infty, \infty) & C: [0, \infty) & D: [2, \infty) & E: \text{None of A, B, C or D} \\
   \hline
   \end{array} \]

A4. [1 mark] Find all x-intercepts of the graph of the function \( y = x^2 - 2 \).
   \[ \begin{array}{|c|}
   \hline
   A: \text{no x-intercepts} & B: x = -2 & C: x = -\sqrt{2} \text{ only} \\
   \hline
   D: x = \sqrt{2} \text{ and } x = -\sqrt{2} & E: x = 0 \\
   \hline
   \end{array} \]

A5. [1 mark] Find an equation of the line with slope 2 which passes through the point (1, -1).
   \[ \begin{array}{|c|}
   \hline
   A: y = -x & B: y = 2x - 3 & C: y = -2x + 1 & D: y = 2x + 3 & E: y = -2x - 1 \\
   \hline
   \end{array} \]

A6. [1 mark] If \( f(x) = x + 2 \) and \( g(x) = \frac{2}{x^2 - 1} \), find \( f(g(x)) \).
   \[ \begin{array}{|c|}
   \hline
   A: \frac{2(x+2)}{x^2 - 1} & B: x + 2 + \frac{2}{x^2 - 1} & C: \frac{2x^2}{x^2 - 1} \\
   \hline
   D: \frac{2}{(x+2)^2 - 1} & E: \frac{4}{x^2 - 1} \\
   \hline
   \end{array} \]

A7. [1 mark] Evaluate \( \lim_{x \to 2} \frac{x^2 - 4}{2 + x} \) if it exists.
   \[ \begin{array}{|c|}
   \hline
   A: 2 & B: -2 & C: 4 & D: -4 & E: \text{does not exist} \\
   \hline
   \end{array} \]

A8. [1 mark] Evaluate \( \lim_{x \to 3} \frac{3-x}{x-3} \) if it exists.
   \[ \begin{array}{|c|}
   \hline
   A: 1 & B: -1 & C: 3 & D: \text{does not exist} & E: \text{None of A, B, C or D} \\
   \hline
   \end{array} \]

A9. [1 mark] Evaluate \( \lim_{x \to -\infty} \frac{x + x^3 + x^5}{1 - x^2 + x^4} \) if it exists.
   \[ \begin{array}{|c|}
   \hline
   A: 1 & B: -1 & C: 0 & D: \infty & E: -\infty \\
   \hline
   \end{array} \]

A10. [1 mark] Find all x-values at which the function \( f(x) = \frac{x - 3}{x^2 - 3x} \) is discontinuous.
    \[ \begin{array}{|c|}
    \hline
    A: x = 3 \text{ only} & B: x = 0 \text{ only} & C: x = -3 \text{ only} \\
    \hline
    D: no value of x & E: x = 3 \text{ and } x = 0 \text{ only} \\
    \hline
    \end{array} \]
A11. [1 mark] Find the slope of the tangent line to the curve \( y = 2x^2 \) at \( x = -1 \).

A: -4 B: 4 C: 1 D: -2 E: 2

A12. [1 mark] Let \( f \) and \( g \) be functions with \( f(1) = 3 \), \( f'(1) = 4 \), \( f'(2) = 5 \), \( g(1) = 2 \), \( g'(1) = 6 \), and \( g'(4) = 7 \). If \( h(x) = f(x)g(x) \), find \( h'(1) \).

A: 24 B: 35 C: 28 D: 26 E: 30

Use the following information to answer questions A13 and A14.

Consider the function \( f(x) = \frac{(x+2)(x-3)}{2x-3x^2} \).

A13. [1 mark] Which of the following gives the equation(s) of all horizontal asymptotes of the graph of \( y = f(x) \)?

A: \( y = \frac{1}{3} \) B: \( x = 0 \) and \( x = \frac{2}{3} \) C: \( y = \frac{1}{2} \)

D: \( x = 0 \) and \( x = \frac{3}{2} \) E: no horizontal asymptotes

A14. [1 mark] Which of the following gives the equation(s) of all vertical asymptotes of the graph of \( y = f(x) \)?

A: \( y = \frac{1}{3} \) B: \( x = 0 \) and \( x = \frac{2}{3} \) C: \( y = \frac{1}{2} \)

D: \( x = 0 \) and \( x = \frac{3}{2} \) E: no vertical asymptotes

Use the following information to answer questions A15 to A20.

Consider the function \( f(x) = x^3(x+4) \) which has \( f'(x) = 4x^2(x+3) \) and \( f''(x) = 12x(x+2) \).

A15. [1 mark] Find all interval(s) on which the function \( f \) is increasing.

A: \( (-\infty, -3) \) only B: \( (-\infty, -3) \) and \( (3, \infty) \) C: \( (-3, 0) \) and \( (0, \infty) \)

D: \( (-4, \infty) \) E: \( (0, \infty) \) only

A16. [1 mark] Find a relative minimum value of the function \( f \), if there is one.

A: -3 B: -27 C: 4 D: 0 E: \( f \) has no relative minimum

A17. [1 mark] Find a relative maximum value of the function \( f \), if there is one.

A: 0 B: 2 C: 4 D: 3 E: \( f \) has no relative maximum

A18. [1 mark] On what interval is the function concave downward?

A: \( (-\infty, 0) \) B: \( (0, \infty) \) C: \( (1, \infty) \) only D: \( (-2, 0) \) E: \( (-\infty, \infty) \)

A19. [1 mark] On what interval(s) is the function concave upward?

A: \( (-\infty, -2) \) and \( (0, \infty) \) B: \( (-\infty, -2) \) only C: \( (0, \infty) \) only

D: \( (-\infty, -4) \) and \( (4, \infty) \) E: \( (-\infty, 0) \)

A20. [1 mark] Which of the following gives all inflection points of the function \( f \)?

A: \( (-2, -16) \) B: \( (0, 0) \) and \( (-2, -16) \) C: \( (0, 0) \) D: \( (-4, 0) \) and \( (-3, 0) \) E: \( (-3, 0) \)
### INSTRUCTOR & COURSE EVALUATION
THE UNIVERSITY OF WESTERN ONTARIO

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**The University of Western Ontario**

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1.4.7 Mathematics 1120

Mathematics 1120A/B - Fundamental Concepts in Mathematics

Primarily for students interested in pursuing a degree in one of the mathematical sciences. Logic, set theory, relations, functions and operations, careful study of the integers, discussion of the real and complex numbers, polynomials, and infinite sets.

Antirequisite(s): Mathematics 2155A/B.

Prerequisite(s): One or more of Ontario Secondary School MCV4U, the former Ontario Secondary School MGA4U, Linear Algebra 1600A/B, or the former Mathematics 017a/b.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 4 lecture hours, 0.5 course.
Mathematics 1120B

2009-2010

INSTRUCTOR: L.E. Renner, Middlesex College, Room 130


PREREQUISITES: One or more of Ontario Secondary School MCV4U, the former Ontario Secondary School MGA4U, Linear Algebra 1600A/B, or the former Mathematics 017a/b.

ANTIREQUISITE: Mathematics 2155A.

COURSE OUTLINE: Mathematics 1120B is an introduction to rigorous mathematical thinking. The main purpose of the course is to teach students to understand mathematics and write mathematical proofs. There will be regular homework assignments to help the students acquire information and develop analytical skills.

TOPICS: logic, proofs, induction, sets, functions, relations, limits, complex numbers, divisibility, polynomials.

EVALUATION OF STUDENT PERFORMANCE:

Grades will be based on assignments and the final exam.

Assignments: 40%

Two One-Hour, In-Class, Midterm Exams: 10% each. February 8 and March 8, 2010

Final Examination: 40% To be scheduled by the Registrar.

IMPORTANT SENATE POLICY:

Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded
student may be removed from the course and it will be deleted from the student's record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites.

**STATEMENT ON ACADEMIC OFFENCES:**

"Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site: http://www.uwo.ca/univsec/handbook/appeals/scholoff.pdf"

"Computer-marked multiple-choice tests and/or exams may be subject to submission for similarity review by software that will check for unusual coincidences in answer patterns that may indicate cheating."

**NEW MEDICAL EXCUSE REGULATIONS:**

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's Office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination” form must be obtained from the Dean's Office immediately.

For further information please see:


A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services.

The form can be found here:

https://studentservices.uwo.ca/secure/medical_document.pdf
Instructions: Answer completely as many questions as you can. Write your proofs and solutions clearly and precisely, using complete sentences, on the booklets provided. No calculators, notes or books are permitted.

Total Marks: [80]

1. [8] Let $A$ and $B$ be sets. Prove that
\[
(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A).
\]

2. [6] Let $n$ be an integer.
   (a) Find the contrapositive of the statement, “If $n$ is positive then $2n$ is even.”
   (b) Find the negation of the statement, “If $n$ is positive then $2n$ is even.”
   (c) Find the converse of the statement, “If $n$ is positive then $2n$ is even.”

3. [8] Which of the following sentences are statements?
   (a) $x^2 + y^2 = 1$.
   (b) Where is the money?
   (c) There is no money in the cookie jar.
   (d) If $x^2 + y^2 = 1$ then $|x| + |y| \neq 0$.

4. [8] Prove by the method of contrapositive. Let $x \in \mathbb{R}$. If $2x^3 - x^2 + 4x - 1 \geq 0$, then $x \geq 0$.

5. (a) [3] Prove by the method of direct proof. The sum, $r + s$, of two rational numbers $r$ and $s$, is a rational number.
   (b) [5] Prove by the method of contradiction. The sum, $x + r$, of an irrational number $x$ and a rational number $r$ is an irrational number.

6. [8] Prove by induction that, for all integers $n > 0$,
\[
1 + 3 + 5 + \ldots + (2n - 1) = n^2.
\]
Make sure that you indicate clearly all the relevant steps of the induction proof.

7. (a) [3] Let $R \subset A \times A$ be a relation. Define “$R$ is an equivalence relation.”
(b) [3] Prove that the function, $f([x]) = [3x]_{21}$, from $\mathbb{Z}_7$ to $\mathbb{Z}_{21}$, is well defined.
(c) [4] Define an equivalence relation on $\mathbb{Z}$ with exactly seven equivalence classes.

8. (a) [4] State the Division Algorithm.
(b) [4] Use the Euclidean Algorithm to find the g.c.d. of 30 and 55.

9. (a) [3] Define what it means for a relation, $R \subseteq A \times B$, to be a function.
(b) [5] Let $f : \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \setminus \{-1\}$ be the function defined by the equation $f(x) = \frac{x}{x^2}$. Prove that $f$ is bijective (i.e. one-to-one and onto).

10. (a) [4] Let $\{a_n\}$ be a sequence of real numbers. Define (using $\epsilon$ and $N$), “$\lim_{n \to \infty} (a_n) = L$”.
(b) [4] Prove that $\lim_{n \to \infty} \left(\frac{1}{2^n} + 1\right) = 1$. 
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<td>5.05</td>
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<tr>
<td>13. Grades Work Promptly</td>
<td>20</td>
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<td>0</td>
<td>2</td>
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<td>6</td>
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<td>14. Good Motivator</td>
<td>20</td>
<td>1</td>
<td>0</td>
<td>5</td>
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<td>15. Overall Effectiveness</td>
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<td>2</td>
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<td>5</td>
<td>3</td>
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<td>1.57</td>
<td>5</td>
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<tr>
<td>16. Course As Learning Experience</td>
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<td>1</td>
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<td>4</td>
<td>5</td>
<td>3</td>
<td>4.96</td>
<td>1.50</td>
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</tbody>
</table>
Mathematics 1225

Mathematics 1120A/B - Fundamental Concepts in Mathematics

Primarily for students interested in pursuing a degree in one of the mathematical sciences.
Logic, set theory, relations, functions and operations, careful study of the integers, discussion of the real and complex numbers, polynomials, and infinite sets.

Antirequisite(s): Mathematics 2155A/B.
Prerequisite(s): One or more of Ontario Secondary School MCV4U, the former Ontario Secondary School MGA4U, Linear Algebra 1600A/B, or the former Mathematics 017a/b.
Corequisite(s):

Pre-or Corequisite(s):
Extra Information: 4 lecture hours, 0.5 course.
THE UNIVERSITY OF WESTERN ONTARIO

LONDON CANADA

DEPARTMENT OF MATHEMATICS

Mathematics 1225B

2009

Math 1225B home page:

NOTE: Many science/technically oriented courses and programs, at both Western and other universities, require a full-year calculus course (such as our Calculus 1000A followed by 1501B). Mathematics 1225B (a half course) may not be appropriate. Furthermore, Mathematics 1225B is not a suitable prerequisite for Calculus 1501B or 1301B.

INSTRUCTORS: P. Milnes, J. Nichols-Barrer; C. Florence (Brescia), S. Kuzmin (Huron), D. Meredith (King's)

TEXTBOOK:  *Calculus with Applications* by Dale Varberg and Walter Fleming (soft cover) published by Prentice Hall.

OR

*Calculus with Applications* by Dale Varberg and Walter Fleming (ISBN 0-13-1108263), published by Prentice Hall. (This hard cover version of the textbook is out of print, but may be available at the Used Book Store.)


(Choose either or both, depending on your preference and whether you have access to them. It is highly recommended.)

PREREQUISITES: One or more of Ontario Secondary School MCV4U, Mathematics 0110A/B, Calculus 1000A/B, 1100A/B, or the former Ontario Secondary School MCB4U.

Note: Unless you have either the prerequisite for this course or written special permission from your Dean to enroll in it, you will be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisite.

ANTIREQUISITES: Calculus 1201A/B, 1301A/B, 1501A/B, Applied Mathematics 1413, the former Mathematics 030.

COURSE OUTLINE: Logarithmic, exponential and trigonometric functions; theory of integration, elementary techniques of integration, applications of integration (e.g., area, volume); functions of several variables (Lagrange Multipliers); improper integrals; differential equations.

WHAT IS EXPECTED OF THE STUDENT?

Students should attend all classes, make a serious effort to understand all course material, and do the homework. The student must assume responsibility for missed classes.

EVALUATION OF STUDENT PERFORMANCE:

There will be two 90-minute multiple choice tests, each worth 25% of the final mark, and a 3-hour multiple choice final examination, worth 50%, during the April examination period. The dates for the 90-minute tests are **Friday, February 6, 2009** and **Friday, March 6, 2009, 7-8:30 pm**.
3. Cheating on a test or examination is a Scholastic Offense, for which penalties are potentially severe. On multiple-choice tests, computer software may be used to check for unusual coincidences in answer patterns that may indicate cheating.

**Faculty of Science Medical Excuse Regulations**

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately. For further information please go to [http://www.uwo.ca/univsec/handbook/appeals/medical.pdf](http://www.uwo.ca/univsec/handbook/appeals/medical.pdf).

A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services. The form can be found at [https://studentservices.uwo.ca/secure/medical_document.pdf](https://studentservices.uwo.ca/secure/medical_document.pdf).
1. Find the value of $3^{2 \log_3 5}$.
   
   | A: 125 | B: 243 | C: 25 | D: 9 | E: 10 |

2. Simplify $\frac{\log_2 3 + \log_2 27}{\log_2 3}$.
   
   | A: $\log_2 27$ | B: 10 | C: 9 | D: 4 | E: 12 |

3. If $f(x) = \ln(x^2 + 1)$, find $f'(1)$.
   
   | A: $-1$ | B: 0 | C: 1 | D: $\frac{3}{2}$ | E: 2 |

4. If $f(x) = x^{(x^2)}$, find $f'(x)$.
   
   | A: $x^2 \cdot x^{(x^2-1)}$ | B: $(2x \ln x)x^{(x^2)}$ | C: $x(2 \ln x + 1)x^{(x^2)}$ | D: $(\ln x + 1)x^{(x^2)}$ | E: $x^{(x^2)}$ |

5. Given that $f(x) = xe^{-x}$, $f'(x) = (1-x)e^{-x}$, and $f''(x) = (x-2)e^{-x}$, find the interval on which $f(x)$ is both decreasing and concave down.
   
   | A: $(-\infty, 1)$ | B: $(-\infty, 2)$ | C: $(1, \infty)$ | D: $(0, 2)$ | E: $(1, 2)$ |

6. Determine the $x$-coordinate of the point on the curve $y = 2x \ln x$ at which the tangent line is parallel to the line $y = 4x + 9$.
   
   | A: 1 | B: $e$ | C: $\frac{1}{e}$ | D: $2e$ | E: $2e^2$ |

7. If $f'(x) = 4x^3 + 2x$ and $f(0) = 5$, then $f(1) =$
   
   | A: 7 | B: 8 | C: 9 | D: 10 | E: 11 |

8. Find $\int (x + 2)^4 \, dx$.
   
   | A: $4(x + 2)^3 + C$ | B: $5(x + 2)^4 + C$ | C: $\frac{(x + 2)^5}{5} + C$ |
   | D: $\frac{(x + 2)^4}{5} + C$ | E: $\frac{2(x + 2)^5}{5} + C$ |

9. If $\frac{3x + 7}{(x + 2)(x + 3)} = \frac{A}{x + 2} + \frac{B}{x + 3}$, find the value of $B$.
   
   | A: $-2$ | B: $-1$ | C: 0 | D: 1 | E: 2 |

10. Evaluate $\int_0^1 \frac{2x}{(x^2 + 1)^3} \, dx$.
     
     | A: $\frac{7}{48}$ | B: $\frac{7}{24}$ | C: $\frac{7}{16}$ | D: $\frac{9}{8}$ | E: $\frac{9}{4}$ |

11. Find the area of the region bounded by the curves $y = e^{-x}$ and $y = 0$ between $x = -2$ and $x = 0$.
    
    | A: $1 - \frac{1}{e^2}$ | B: $1 - \frac{1}{e}$ | C: $e^2 - 1$ | D: $e - 1$ | E: 2 |
12. Find the area of the region bounded by the curves \( y = x^2 \) and \( y = x + 2 \).

| A: \( \frac{9}{2} \) | B: \( \frac{7}{6} \) | C: \( \frac{10}{3} \) | D: 3 | E: \( \frac{15}{2} \) |

13. Which of the following gives the volume when the shaded region is rotated about the \( x \)-axis?

| A: \( \pi \int_0^1 e^{4x} \, dx \) | B: \( \pi \int_0^1 (e^2 - e^{2x})^2 \, dx \) | C: \( \pi \int_0^1 (e^4 - e^{4x}) \, dx \) |

| D: \( \pi \int_0^{e^2} \left( \frac{1}{2} \ln y \right)^2 \, dy \) | E: \( \pi \int_0^{e^2} \frac{1}{2} \ln y \, dy \) |

14. Which of the following gives the volume when the shaded region is rotated about the \( y \)-axis?

| A: \( \pi \int_0^1 e^{4x} \, dx \) | B: \( \pi \int_0^1 (e^2 - e^{2x})^2 \, dx \) | C: \( \pi \int_0^1 (e^4 - e^{4x}) \, dx \) |

| D: \( \pi \int_1^{e^2} \left( \frac{1}{2} \ln y \right)^2 \, dy \) | E: \( \pi \int_1^{e^2} \frac{1}{2} \ln y \, dy \) |

15. Evaluate the improper integral \( \int_0^{\infty} \frac{x}{x^2 + 5} \, dx \).

| A: \( \frac{1}{5} \) | B: \( \frac{1}{10} \) | C: \( -\frac{1}{5} \) | D: \( -\frac{1}{10} \) | E: diverges |

16. If \( f(x, y, z) = xy \ln(xy - z) \), find \( f_x(2, 1, 1) \).

| A: 0 | B: 1 | C: 2 | D: \( \ln 2 \) | E: \( \frac{1}{2} \) |

17. Find \( f_{xy}(x, y) \) where \( f(x, y) = \ln(x^2y^2) + \sqrt{x}y^2 \).

| A: \( xy \sqrt{x} \) | B: \( y \sqrt{x} \) | C: \( \frac{y}{\sqrt{x}} \) | D: \( \frac{6}{xy^2} + \frac{y}{\sqrt{x}} \) | E: \( -\frac{y}{\sqrt{x}} \) |

18. How many critical points does \( f(x, y) = xy - x^2y - xy^2 \) have?

| A: 0 | B: 1 | C: 2 | D: 3 | E: 4 |

Use the following information to answer questions 19 and 20.

\[
f(x, y) = x^2y^2 - 2xy^2 - 2x^2y \\
f_x = 2xy^2 - 2y^2 - 4xy, \quad f_y = 2x^2y - 4xy - 2x^2 \\
f_{xx} = 2y^2 - 4y, \quad f_{yy} = 2x^2 - 4x, \quad f_{xy} = 4xy - 4y - 4x
\]

19. Consider the point \( (3, 0) \). Which of the following statements is true?
20. Consider the point (3, 3). Which of the following statements is true?

<p>| | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>A</td>
<td>(3, 3) is not a critical point of ( f(x, y) ).</td>
</tr>
<tr>
<td>B</td>
<td>There is a local maximum at (3, 3).</td>
</tr>
<tr>
<td>C</td>
<td>There is a local minimum at (3, 3).</td>
</tr>
<tr>
<td>D</td>
<td>There is a saddle point at (3, 3).</td>
</tr>
</tbody>
</table>

21. Suppose we use Lagrange’s method to maximize \( f(x, y, z) = x^2 + y^2 + 2z \) subject to the constraint \( x + y + z^2 = 2 \). What system of equations must be solved?

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<tbody>
<tr>
<td>A</td>
<td>( x^2 + y^2 + 2z + \lambda(x + y + z^2 - 2) = 0 )</td>
</tr>
</tbody>
</table>
| B | \( 2x + \lambda = 0 \\
2y + \lambda = 0 \\
2 + 2\lambda z = 0 \\
x + y + z^2 = 0 \) |
| C | \( x + y + z^2 - 2 + \lambda(x^2 + y^2 + 2z) = 0 \) |
| D | \( 1 + 2\lambda x = 0 \\
1 + 2\lambda y = 0 \\
2z + 2\lambda = 0 \\
x^2 + y^2 + 2z = 0 \) |
| E |   \\
2x + \lambda = 0 \\
2y + \lambda = 0 \\
1 + \lambda z = 0 \\
x + y + z^2 - 2 = 0 |

22. Suppose you are to design a rectangular box with no top, of length \( l \), width \( w \) and height \( h \), with the minimum surface area. Assume also that the volume of the box must be 1000 cm\(^3\). Set this problem using Lagrange’s method.

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<tbody>
<tr>
<td>A</td>
<td>( F(l, w, h, \lambda) = lw + 2lh + 2wh + \lambda(lw) )</td>
</tr>
<tr>
<td>B</td>
<td>( F(l, w, h, \lambda) = lw + 2lh + 2wh + \lambda(l + w + h - 1000) )</td>
</tr>
<tr>
<td>C</td>
<td>( F(l, w, h, \lambda) = lwh + \lambda(lw + 2lh + 2wh - 1000) )</td>
</tr>
<tr>
<td>D</td>
<td>( F(l, w, h, \lambda) = lw + 2lh + 2wh + \lambda(lwh - 1000) )</td>
</tr>
<tr>
<td>E</td>
<td>( F(l, w, h, \lambda) = 2lw + 2lh + 2wh + \lambda(lwh - 1000) )</td>
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</table>

23. In using Lagrange’s method to solve a particular constrained optimization problem, the following system of equations needs to be solved:

\[
\begin{align*}
4 + 2\lambda x &= 0 \\
8 + 2\lambda y &= 0 \\
x^2 + y^2 &= 5
\end{align*}
\]

Find all possible solutions to this system.

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<tbody>
<tr>
<td>A</td>
<td>(2, 1) and (-2, -1)</td>
</tr>
<tr>
<td>B</td>
<td>(1, 2) and (-1, -2)</td>
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<tr>
<td>C</td>
<td>(-2, 1) and (2, -1)</td>
</tr>
<tr>
<td>D</td>
<td>(1, -2) and (-1, 2)</td>
</tr>
<tr>
<td>E</td>
<td>(2, 1), (1, 2), (-1, -2) and (-2, -1)</td>
</tr>
</tbody>
</table>
24. In using Lagrange's method to find the maximum value of \( f(x, y) = xy + 12 \) subject to the constraint \( x^2 + 4y^2 = 32 \), we find the points \((4, 2), (4, -2), (-4, 2)\) and \((-4, -2)\) and determine that the maximum value of \( f \) subject to the constraint is

| A: 12 | B: 24 | C: 40 | D: 20 | E: 32 |

25. If \( f(x, y, z) = ze^{xy} \sin x \ln y \), find \( f\left(\frac{\pi}{2}, \sqrt{e}, 1\right) \).

| A: 1 | B: 2 | C: 1 | D: \pi | E: e |

26. If \( f(x) = x^2 \sin x \) then \( f'(\pi) = \)

| A: \pi^2 | B: -\pi^2 | C: 2\pi | D: -2\pi | E: 0 |

27. If \( f(x) = \sin^2 x + \cos^2 x \) then \( f'(\frac{\pi}{3}) = \)

| A: -2 | B: 2 | C: -1 | D: 1 | E: 0 |

28. If \( f(x) = \tan 4x \), then \( f'(x) = \)

| A: \sec^2 4x | B: 4 \sec^2 4x | C: 16 \sec^2 4x | D: \frac{1}{4} \sec^2 4x | E: \tan 4x |

29. If \( f(x) = \cos(2\pi x) \), then \( f''(x) = \)

| A: -\sin(2\pi x) | B: 2\pi \cos(2\pi x) | C: -2\pi \cos(2\pi x) | D: 4\pi^2 \cos(2\pi x) | E: -4\pi^2 \cos(2\pi x) |

30. If \( f(x) = \sec^2 x \) find the slope of the tangent line to the graph of \( y = f(x) \) at \( x = \frac{\pi}{4} \).

| A: 4 | B: 1 | C: \frac{\sqrt{2}}{2} | D: -1 | E: 2 |

31. \( \int \sec^2 x \, dx = \)

| A: \frac{\sec^3 x}{3} + C | B: 2 \sec^2 x \tan x + C | C: \tan^2 x \sec x + C |

| D: \tan x + C | E: \ln(\cos^2 x) + C |

32. \( \int \frac{\sin \left(\frac{1}{x}\right)}{x^2} \, dx = \)

| A: -\cos \frac{1}{x} + C | B: -\cos \left(\frac{1}{x}\right) + C | C: \cos \left(\frac{1}{x}\right) + C |

| D: \cos \frac{1}{x} + C | E: \cos \left(\frac{1}{x}\right) + C |

33. \( \int \frac{\sin x}{1 - \cos x} \, dx = \)

| A: \frac{\cos x}{1 - \cos x} + C | B: \ln |1 - \cos x| + C | C: \cot x + C |

| D: -\ln |\cos x| + C | E: \ln |\tan x| + C |

34. The integral \( \int x \cos x \, dx \) can be written as
5. \[ x \sin x - \int \sin x \, dx \]
6. \[ \frac{1}{2} x^2 \cos x - \int \sin x \, dx \]
7. \[ x \cos x - \int \sin x \, dx \]
8. \[ x \sin x + \frac{1}{2} \int x^2 \sin x \, dx \]
9. \[ \frac{1}{2} x^2 \cos x - \frac{1}{2} \int x^2 \sin x \, dx \]

35. \[ \int_0^{\sqrt{\pi}} x \sin(x^2) \, dx = \]
   A: \( \frac{\pi}{2} \)
   B: 1
   C: 0
   D: 2
   E: -1

36. \[ \int_0^{\frac{\pi}{4}} \tan^2 x \sec^2 x \, dx = \]
   A: \( -\frac{1}{4} \)
   B: 1
   C: 0
   D: \( \frac{1}{3} \)
   E: \( \frac{1}{2} \)

37. If \( \frac{dy}{dt} = 2t \) and \( y(0) = 4 \), find \( y(1) \).
   A: 5
   B: 6
   C: \( 2e^4 \)
   D: \( 4e^2 \)
   E: 0

38. If \( \frac{dy}{dt} = 2y \) and \( y(0) = 4 \), find \( y(1) \).
   A: 5
   B: 6
   C: \( 2e^4 \)
   D: \( 4e^2 \)
   E: 0

39. If \( \frac{dy}{dt} = ky \), \( y(0) = 2 \) and \( y(1) = 3 \), find \( k \).
   A: \( \ln 6 \)
   B: \( \ln 3 \)
   C: \( \ln 2 \)
   D: \( \ln \left(\frac{2}{3}\right) \)
   E: \( \ln \left(\frac{3}{2}\right) \)

40. Solve for \( y \) if \( \frac{dy}{dx} = \frac{x^2}{y^2} \).
   A: \( y = x + C \)
   B: \( y = x^2 + C \)
   C: \( y = x^3 + C \)
   D: \( y = \sqrt{x^3 + C} \)
   E: \( y = \sqrt{x^3 + C^2} \)

41. Solve for \( y \) if \( \frac{dy}{dx} = \frac{3 \cos x}{e^y} \).
   A: \( y = \ln(3 \sin x) + C \)
   B: \( Ce^y = \sin x \)
   C: \( y = \ln(3 \sin x + C) \)
   D: \( y = \ln(3 \cos x + C) \)
   E: \( y = C \sin x \)

42. Which of the following is an integrating factor that can be used to solve \( \frac{dy}{dx} - y = xe^x \cos x \)?
   A: \( -x \)
   B: \( e^{-x} \)
   C: \( e^x \)
   D: \( x \)
   E: \( xe^x \)

43. Which of the following is an integrating factor that can be used to solve \( \frac{dy}{dx} + \frac{y}{x} = x^2 + 1 \)?
   A: \( x \)
   B: \( \frac{1}{x} \)
   C: \( x^2 \)
   D: \( \frac{1}{x^2} \)
   E: \( e^x \)

44. Find \( y(-1) \) where it is known that \( \frac{dy}{dx} + 3x^2y = 3x^2 \) and \( y(0) = 2 \) using the integrating factor \( I(x) = e^{x^3} \).
45. If \( \frac{dy}{dt} = ky \) and it is known that \( y(4) = 4y(2) \), what is the value of \( k? \)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln 4 )</td>
<td>2</td>
<td>4</td>
<td>( \ln 2 )</td>
<td>( \frac{1}{2} \ln 2 )</td>
</tr>
</tbody>
</table>

46. A certain curve passes through the point \( \left( \frac{\pi}{2}, 0 \right) \). The slope of the tangent line to the curve at the point \((x, y)\) is equal to \( \frac{\cos x}{\sin y} \). Find the equation for this curve.

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<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin x + \cos y = \frac{\pi}{2} )</td>
<td>( \sin x + \cos y = 2 )</td>
<td>( \cos y = -\sin x )</td>
<td>( \cot \left( \frac{x}{y} \right) = 1 )</td>
<td>( \cos y - \sin x = 1 )</td>
</tr>
</tbody>
</table>

47. The population \( y(t) \) of a certain city increases exponentially over time \((t)\), according to the differential equation \( \frac{dy}{dt} = ky \). Ten years ago the population was 1 million and this year the population is 2 million. After how many years from now will the population be 10 million?

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{10 \ln 2}{\ln 5} )</td>
<td>( \frac{\ln 10}{\ln 2} )</td>
<td>( \frac{10 \ln 5}{\ln 2} )</td>
<td>( \frac{2 \ln 10}{\ln 2} )</td>
<td>( \frac{10 \ln 5}{\ln 10} )</td>
</tr>
</tbody>
</table>

48. A radioactive element decays with a half-life of 100 years. If we have 10 grams of this element today, how many grams were there 50 years ago?

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<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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</thead>
<tbody>
<tr>
<td>( \frac{40}{3} )</td>
<td>20</td>
<td>( 5 \sqrt{\frac{1}{2}} )</td>
<td>( 10 \sqrt{\frac{1}{2}} )</td>
<td>( 10 \sqrt{2} )</td>
</tr>
</tbody>
</table>

49. A tank contains 100 litres of pure water. A salt solution containing 100 grams of salt per litre is allowed to flow into the tank at the rate of 10 litres per minute. The well-stirred solution flows out of the tank also at 10 litres per minute. Let \( y(t) \) equal the number of grams of salt in the tank \( t \) minutes after the solution was allowed to flow into it. Construct the mathematical model needed to find \( y(t) \).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{dy}{dt} = 1000 - \frac{y}{10}, \quad y(0) = 10 )</td>
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50. On a planet where the acceleration due to gravity is 24 metres per second per second, a steel ball is dropped from a height of 1700 metres. Find the height of the ball in metres after 10 seconds.

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Average of 1 to 14: 5.43, Standard Deviation: 1.09, Median: 5

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Average of 1 to 14: 4.34, Standard Deviation: 1.77, Median: 4
## INSTRUCTOR & COURSE EVALUATION

### THE UNIVERSITY OF WESTERN ONTARIO

| Instructor Name | Teaching Faculty | Teaching Department | Subject | Course Number | Section | Year | Term | Enrolment | Responses |
|-----------------|------------------|---------------------|---------|---------------|---------|------|------|-----------|-----------|-----------|
| Olds, Victoria  | Science          | Mathematics         | Mathematics | 1225B        | 001     | 2008-09 | Summer | 60        | 28        |

### Questions and Responses

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### Additional Information

- **Percent of Classes Attended**
  - 90% or more: 21
  - 70% to 89%: 2
  - 50% to 69%: 1
  - 20% to 49%: 0

- **Expected Grade**
  - A: 9
  - B: 11
  - C: 8
  - D: 0

- **Course Status**
  - Required: 24
  - Optional: 1
  - Total: 25

- **Initial Level of Enthusiasm**
  - High: 9
  - Medium: 10
  - Low: 9
  - Total: 28

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Mathematics 1228A/B - Methods of Finite Mathematics

Permutations and combinations; probability theory. This course is intended primarily for students in the Social Sciences, but may meet minimum requirements for some Biological or Basic Medical Sciences modules.

Antirequisite(s): Mathematics 2124A/B, 2155A/B, Statistical Sciences 2035, 2141A/B, 2857A/B, the former Mathematics 031, the former Statistical Sciences 2657A.

Prerequisite(s): One or more of Ontario Secondary School MCV4U, MHF4U, MDM4U, Mathematics 0110A/B, 1225A/B, 1229A/B, the former Mathematics 017a/b, the former Ontario Secondary School MGA4U, MCB4U.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
Mathematics 1228B

INSTRUCTORS:
J. Adcock (King's), O. Flint, C. Florence (Brescia), S. Joyner, S. Kuzmin (Huron), J. Minac, V. Olds.

TEXTBOOK:

PREREQUISITES:
One or more of Ontario Secondary School MCV4U, MHF4U, MDM4U, Mathematics 0110A/B, 1225A/B, 1229A/B, the former Mathematics 017a/b, the former Ontario Secondary School MGA4U, MCB4U.

ANTIREQUISITES:

COURSE OUTLINE:
Techniques of counting, probability, discrete and continuous random variables.

EVALUATION OF STUDENT PERFORMANCE:
There will be 2 term tests, each 90 minutes in length, held 7:00 - 8:30 p.m. on Friday, January 30, and on Friday, March 13. Each test will count for 25% of the student's final grade. (Locations of these tests will be announced.)

The Final Exam will be 3 hours in length, covering all of the course material, and will count for 50%. The exam will be scheduled during the April Exam Period.

COURSE WEB SITE:
Information such as test rooms, etc. will be posted on the course website in WebCT. To access the course web site go to http://webct.uwo.ca and choose The University of Western Ontario. Use your UWO ITS username and password (i.e. same as for UWO email) to login. Various useful supplemental materials, such as practice tests and solutions to the homework exercises, are posted on the web site. In addition, there are discussion boards on which students may post questions.

IMPORTANT SENATE POLICY:
Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded and that all course prerequisites have been successfully completed. If the student does not have the prerequisites for a course, and does not have written special permission from his or her Dean to enroll in the course, the student may be removed from the course and it will be deleted from the student's record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites.
STATEMENT ON ACADEMIC OFFENCES:
Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site: http://www.uwo.ca/univsec/handbook/appeals/scholoft.pdf
Computer-marked multiple-choice tests and/or exams may be subject to submission for similarity review by software that will check for unusual coincidences in answer patterns that may indicate cheating.

NEW MEDICAL EXCUSE REGULATIONS:
If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's Office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with his or her instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately. For further information please see: http://www.uwo.ca/univsec/handbook/appeals/medical.pdf

A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services. The form can be found here: https://studentservices.uwo.ca/secure/medical_document.pdf
A1. [1 mark] This year marks the 200th anniversary of the birth of Louis Braille, who invented the Braille alphabet which is used by blind and visually impaired people. In the Braille alphabet, letters and other characters are encoded using dots within a 6-dot grid (two columns of three dots). Each of the 6 dots may be raised into a bump, or left flat. The characters are read by touch. For instance, the letter “a” is formed by raising just the top dot in the left-hand column, while the letter “z” is formed by raising the top and bottom dots in the left-hand column and the middle and bottom dots in the right-hand column. Using (only) Braille’s 6-dot grid, raising some, all, or none of the dots, how many different characters can be represented?

A: 6!
B: \(\frac{6!}{2!}\)
C: \(\binom{6}{2}\)
D: \(2^6\)
E: \(6^2\)

A2. [1 mark] See question A1. How many different characters can be represented with exactly two of the dots raised?

A: 4
B: 8
C: 15
D: 30
E: 60

A3. [1 mark] See question A1. How many different characters can be represented with at least one of the dots in the left-hand column raised?

A: 24
B: 32
C: 48
D: 56
E: 64

A4. [1 mark] Find the number of permutations of the letters in the word DESSERTS if the three S’s must be together.

A: \(\frac{8!}{3!2!}\)
B: \(\frac{6!}{2!}\)
C: 3!6!
D: 3!5!
E: 5!

A5. [1 mark] Pete’s Pizza has 25 different toppings that can be used when ordering a pizza. In how many ways is it possible to order a pizza from Pete’s Pizza which has at least three but not more than five toppings?

A: \(\frac{25!}{3!4!5!}\)
B: \(\binom{25}{3}\binom{22}{4}\binom{18}{5}\)
C: \(\binom{25}{3}\binom{25}{4}\binom{25}{5}\)
D: \(\binom{25}{3} + \binom{22}{4} + \binom{18}{5}\)
E: \(\binom{25}{3} + \binom{25}{4} + \binom{25}{5}\)

A6. [1 mark] There are twelve different books to be distributed among three students, Jim, Frank and Dianne. In how many ways can this be done if one student must receive exactly two books, one student exactly four books and one student exactly six books, but it has not been decided which student will receive which number of books?

A: \(3!\binom{12}{2}\binom{4}{6}\)
B: \(\binom{12}{2}\binom{4}{6} \div 3!\)
C: \(\binom{12}{2}\binom{10}{4}\binom{6}{6}\)
D: \(\binom{12}{2} + \binom{12}{4} + \binom{12}{6}\)
E: \(\binom{12}{2} + \binom{10}{4} + \binom{6}{6}\)

A7. [1 mark] In how many ways can all of ten identical pencils be distributed among three students?

A: \(10^3\)
B: \(3^{10}\)
C: \(\binom{10}{3}\)
D: \(\binom{12}{3}\)
E: \(\binom{12}{10}\)
A8. [1 mark] In how many ways can all of ten different pencils be distributed among three students?

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<tbody>
<tr>
<td>A: $10^3$</td>
<td>B: $3^{10}$</td>
<td>C: $\binom{10}{3}$</td>
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<tr>
<td>D: $\binom{12}{3}$</td>
<td>E: $\binom{12}{10}$</td>
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A9. [1 mark] An experiment consists of tossing an ordinary coin ten times. Which of the following are possible sample spaces for this experiment?

(i) {more heads than tails, fewer heads than tails}
(ii) {at least four heads, at least six tails}
(iii) {no heads, at least one head}
(iv) {at most four heads, at most five tails}

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<table>
<thead>
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<tbody>
<tr>
<td>A: (i) and (ii) only</td>
<td>B: (i) and (iv) only</td>
</tr>
<tr>
<td>C: (ii) and (iii) only</td>
<td>D: (iii) and (iv) only</td>
</tr>
<tr>
<td>E: (ii), (iii) and (iv) only</td>
<td></td>
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A10. [1 mark] Anne, Bob and Carole are candidates for student council president. Anne is four times as likely to win as Bob, but only half as likely to win as Carole. Find the probability that Carole wins.

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<thead>
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<tbody>
<tr>
<td>A: $\frac{9}{14}$</td>
<td>B: $\frac{8}{13}$</td>
<td>C: $\frac{7}{12}$</td>
</tr>
<tr>
<td>D: $\frac{6}{11}$</td>
<td>E: $\frac{5}{9}$</td>
<td></td>
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</tbody>
</table>

A11. [1 mark] Let $E$ and $F$ be events of an experiment with sample space $S$. If $\Pr[E] = .5$, $\Pr[F] = .7$ and $\Pr[E^c \cap F^c] = .2$, find $\Pr[E \cap F]$.

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<td>B: .2</td>
<td>C: .3</td>
<td>D: .4</td>
<td>E: .5</td>
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</table>

A12. [1 mark] Let $A$ and $B$ be two events defined on a sample space $S$, with $\Pr[A] = .3$ and $\Pr[B] = .5$. Which of the following statements are true?

(i) If $A$ and $B$ are mutually exclusive events then $\Pr[A \cap B] = 0$.
(ii) If $A$ and $B$ are independent events then $\Pr[A \cap B] = .15$.
(iii) If $\Pr[B|A] = .7$ then $\Pr[A \cap B] = .21$.

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<td>B: (i) and (iii) only</td>
<td>C: (ii) and (iii) only</td>
</tr>
<tr>
<td>D: none of them</td>
<td>E: all of them</td>
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A13. [1 mark] All of the letters in the word CABIN are randomly put in a line. What is the probability that the letters are in alphabetical order?

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</thead>
<tbody>
<tr>
<td>A: $\frac{4}{5}$</td>
<td>B: $\frac{1}{5}$</td>
<td>C: $\frac{1}{5!}$</td>
<td>D: $\frac{5}{26}$</td>
<td>E: $\frac{21}{26}$</td>
</tr>
</tbody>
</table>

Use the following probability tree in questions A14, A15 and A16.

```
G

.9

H

H^c

G^c

.7

H

H^c
```


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<tbody>
<tr>
<td>A: .1</td>
<td>B: .18</td>
<td>C: .2</td>
<td>D: .25</td>
<td>E: .9</td>
</tr>
</tbody>
</table>

A15. [1 mark] Find $\Pr[H|G]$.

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<tbody>
<tr>
<td>A: .1</td>
<td>B: .18</td>
<td>C: .2</td>
<td>D: .25</td>
<td>E: .9</td>
</tr>
</tbody>
</table>
A16. [1 mark] Find \( \Pr[H] \).

- A: .1
- B: .18
- C: .2
- D: .25
- E: .9

A17. [1 mark] An ordinary six-sided die is rolled 10 times. What is the probability of getting a one at most once in the ten tosses?

\[
\begin{array}{ccc}
\text{A} & = & 1 - \left(\frac{5}{6}\right)^{10} \\
\text{B} & = & \binom{10}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 \\
\text{C} & = & \left(\frac{5}{6}\right)^{10} \\
\text{D} & = & \binom{10}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 + \left(\frac{5}{6}\right)^{10} \\
\text{E} & = & \text{none of A, B, C, D}
\end{array}
\]

A18. [1 mark] On the London-Toronto shuttle, each passenger is offered one complimentary beverage. A flight attendant has noticed that everyone does accept a beverage and that the probability of a passenger having coffee is .6, having tea is .3 and having a juice drink is .1. A shuttle plane is carrying 40 passengers. What is the probability that there will be 20 passengers having coffee, 5 having tea and 15 having a juice drink?

- A: \( \binom{40}{20} \binom{20}{5} \binom{15}{15} \cdot (.6)^{20} \cdot (.3)^5 \cdot (.1)^{15} \)
- B: \( \left(\frac{40}{20} \cdot \frac{5}{15}\right) \cdot (.6)^{20} + (.3)^5 + (.1)^{15} \)
- C: \((.6)^{20} + (.3)^5 + (.1)^{15}\)
- D: \((.6)^{20} \cdot (.3)^5 \cdot (.1)^{15}\)
- E: none of A, B, C, D

A19. [1 mark] Jerry has two boxes of toys. The first box contains 3 trucks and 7 cars; the second box contains 6 trucks and 3 cars. On Monday, Jerry selected one of the toys at random from the first box and placed it in the second box. On the following day, Tuesday, he selected a toy at random from the second box. Given that the toy he selected on Tuesday was a car, what is the probability that the toy he selected on Monday was also a car?

- A: \( \frac{28}{37} \)
- B: \( \frac{7}{30} \)
- C: \( \frac{7}{11} \)
- D: \( \frac{10}{19} \)
- E: none of A, B, C, D

A20. [1 mark] \( X \) is a discrete random variable which assumes only integer values. \( F(x) \) is the cumulative distribution function (cdf) associated with \( X \), and it is known that \( F(5) = 0.73 \), \( F(6) = 0.79 \), \( F(7) = 0.86 \) and \( F(8) = 0.91 \). Find \( \Pr[6 \leq X \leq 7] \).

- A: .07
- B: .13
- C: .18
- D: .21
- E: .27


- A: .07
- B: .13
- C: .18
- D: .21
- E: .27

A22. [1 mark] Discrete random variable \( X \) has the probability distribution function (pdf) shown below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \Pr[X = x] )</th>
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<tbody>
<tr>
<td>-1</td>
<td>1/2</td>
</tr>
<tr>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td>3</td>
<td>1/4</td>
</tr>
</tbody>
</table>

Find \( E(X) \).

- A: \( \frac{1}{4} \)
- B: \( \frac{1}{2} \)
- C: \( \frac{11}{4} \)
- D: 3
- E: 9

A23. [1 mark] See question A22. Find \( E(X^2) \).
A24. [1 mark] Find $E(XY)$ where the two random variables $X$ and $Y$ are defined on the same sample space $S = \{t_1, t_2, t_3\}$ as indicated in the table below.

<table>
<thead>
<tr>
<th>$S$</th>
<th>Probability</th>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
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</tr>
<tr>
<td>$t_2$</td>
<td>.3</td>
<td>0</td>
<td>-10</td>
</tr>
<tr>
<td>$t_3$</td>
<td>.4</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

A: 1 B: 10 C: 20 D: 40 E: 100

Use the following information for questions A25, A26 and A27.

Let $X$ and $Y$ be independent random variables defined on the same sample space $S$ with $E(X) = 2$, $V(X) = 9$, $V(Y) = 4$.

A25. [1 mark] Find $E(X^2)$.

A: 4 B: 5 C: 7 D: 11 E: 13

A26. [1 mark] Find $\sigma(-2X + 3)$.

A: 4 B: 6 C: 12 D: 18 E: 21

A27. [1 mark] Find $V(2X - Y)$.

A: 12 B: 32 C: 40 D: 8 E: 22

A28. [1 mark] Discrete random variable $X$ has possible values 1,3,5,7,9 and 11. Continuous random variable $Y$ is known to be a good approximation for $X$. Which of the following is an approximation of $\Pr[3 < X < 9]$?


In the following questions, $Z$ is the standard normal random variable. Use the table provided at the back of the exam paper for questions A29 through A35.


A: .0668 B: .4332 C: .4568 D: .5668 E: .9332

A30. [1 mark] Find $\Pr[1.0 < Z < 1.9]$.

A: .1300 B: .2600 C: .3700 D: .4030 E: .8159


A: .0150 B: .4850 C: .5150 D: .6950 E: .9850

A32. [1 mark] Find $\Pr[-1.65 < Z < 0]$.

A: .9505 B: .4505 C: .0495 D: .9010 E: 1.65

A33. [1 mark] $X$ is a normal random variable with mean $\mu = 20$ and standard deviation $\sigma = 10$. Find $\Pr[15 \leq X \leq 30]$.

A: .1587 B: .3085 C: .5328 D: .1498 E: .8413

A34. [1 mark] If $\Pr[Z > k] = .67$, what is the value of $k$?

A: .7486 B: 0.33 C: -0.33 D: 0.44 E: -0.44
A35. [1 mark] Let $X$ be a normal random variable with mean $\mu = 8$ and standard deviation $\sigma = 2$. If $\Pr[X < k] = 0.9332$, what is the value of $k$?

<table>
<thead>
<tr>
<th></th>
<th>A: 1.5</th>
<th>B: 3</th>
<th>C: 5</th>
<th>D: 5.5</th>
<th>E: 11</th>
</tr>
</thead>
</table>

**PART B (15 marks)**

**SHOW YOUR WORK**

B1. [3 marks] In an art class of 35 students, each student has to use at least one of the colours red (R), yellow (Y) or blue (B) for a painting project. 25 use red, 21 use yellow, 27 use blue, 13 use red and yellow, 20 use red and blue, 1 uses red only. Complete the counting tree below, showing the appropriate numbers on all the branches.

B2. [3 marks] In Pleasantville, the mornings are sunny (S) 90% of the time. Sam always takes an umbrella (U) when the morning is not sunny. When the morning is sunny he takes an umbrella only 5% of the time.

(a) Fill in the probabilities on all the branches in the probability tree below.

(b) If Sam takes his umbrella, what is the probability that the morning is sunny? **Do not simplify your answer.**

B3. [2 marks] A bag contains 5 red marbles and 2 green marbles. Andrew draws one marble at a time, selected at random and without replacement, until he draws a red marble. Once he draws a red marble he stops drawing marbles. Let the random variable $X$ be the number of draws that Andrew makes.

Make a table for the probability distribution function (pdf) for $X$. You do not need to simplify your answers. Show your work.
B4. \[2 \text{ marks}\] \(X\) is a normal random variable with mean \(\mu = 7\) and \(\Pr[X \leq 10] = .7257\). Find the standard deviation \(\sigma\). Show your work.

B5. \[5 \text{ marks}\] A coin is loaded so that on each toss the probability of heads is \(\frac{1}{3}\) and the probability of tails is \(\frac{2}{3}\). This coin is tossed 18 times. Let the random variable \(X\) be the number of heads obtained in the 18 tosses.
(a) Find \(E(X)\).
(b) Find \(\sigma(X)\).
(c) Use a normal approximation and the table provided at the back of the exam paper to find \(\Pr[X \leq 8]\). Show your work.

The Cumulative Distribution Table for the Standard Normal Random Variable \(Z\)

<table>
<thead>
<tr>
<th>(k)</th>
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<th>.02</th>
<th>.03</th>
<th>.04</th>
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## INSTRUCTOR & COURSE EVALUATION

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### Course Status

- **Required**: 49
- **Optional**: 22
- **Total**: 71

### Initial Level of Enthusiasm

- **High**: 5
- **Medium**: 32
- **Low**: 35
- **Total**: 72

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### INSTRUCTOR & COURSE EVALUATION

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# 118
# INSTRUCTOR & COURSE EVALUATION

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<td>14. Good Motivator</td>
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<th>50% to 69%</th>
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<th>Total Course Status</th>
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<tbody>
<tr>
<td>62</td>
<td>46</td>
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</table>
Mathematics 1229A/B - Methods of Matrix Algebra

Matrix algebra including vectors and matrices, linear equations, determinants. This course is intended primarily for students in the Social Sciences, but may meet minimum requirements for some Biological or Basic Medical Sciences modules.


Prerequisite(s): One or more of Ontario Secondary School MCF3M, MCR3U, or equivalent.

Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 1229A
Methods of Matrix Algebra
Fall, 2009

Math 1229A home page:

Instructors
P. Milnes, D. Barnes, J. Morton, P. Oman
C. Florence (Brescia)
S. Kuzmin (Huron)
D. Meredith (King's)

This course is intended primarily for students in the Social Sciences, but may
meet minimum requirements for some Biological or Basic Medical Sciences modules.


PREREQUISITES: One or more of Ontario Secondary School MCF3M, MCR3U, or equivalent.

Note: Unless you have either the prerequisite(s) for this course or written special
permission from your Dean to enroll in it, you will be removed from this course and
it will be deleted from your record. This decision may not be appealed. You will receive
no adjustment to your fees in the event that you are dropped from a course for failing to
have the necessary prerequisites.

ANTIREQUISITES: Applied Mathematics 1411A/B, 2811B, Linear Algebra 1600A/B,
Mathematics 2120A/B, 2155A, 2211B, the former Mathematics 030, 031, 203b.

COURSE OUTLINE: Vectors in \( \mathbb{R}^n \): Equations of lines and planes; Matrix Algebra: Multiplication, Inverses;
Linear Equations; Determinants.

EVALUATION OF STUDENT PERFORMANCE:

| Term Test #1: Thursday, October 8, 2009, 7-8:30 pm. | 25% |
| Term Test #2: Friday, November 6, 2009, 7-8:30 pm. | 25% |
| Final Examination in December: Scheduled by the Registrar (3 hours) | 50% |

Notes: 1. The tests and examination are multiple choice.
2. Calculators are not allowed on tests and exams.
3. Cheating on a test or examination is a Scholastic Offense, for which penalties
   are potentially severe. On multiple-choice tests, computer software may be used
to check for unusual coincidences in answer patterns that may indicate cheating.

Faculty of Science Medical Excuse Regulations

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If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately. For further information please go to http://www.uwo.ca/univsec/handbook/appeals/medical.pdf.

A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services. The form can be found at https://studentservices.uwo.ca/secure/medical_document.pdf.
1. Find $2\mathbf{u} + \mathbf{v}$.

| A: (6, 1, 0) | B: (7, −1, −1) | C: (11, 4, 1) | D: (7, −7, −1) | E: (−3, −7, −3) |

2. Find $\mathbf{u} \cdot \mathbf{v}$.

| A: −2 | B: −1 | C: 0 | D: 1 | E: 2 |

3. Find $\mathbf{u} \times \mathbf{v}$.

| A: (−1, 6, −13) | B: (1, 6, 13) | C: (−1, −6, −13) | D: (−1, −6, 13) | E: (−1, −6, 13) |

4. Which one of the following describes a line passing through the point $(-1, 3)$ and parallel to the line $5x - 3y = 7$?

| A: $(x, y) = (-1, 3) + t(5, -3)$ | B: $(x, y) = (3, 5) + t(-1, 3)$ | C: $(x, y) = (5, -3) + t(-1, 3)$ | D: $(x, y) = (-1, 3) + t(-3, 5)$ | E: $(x, y) = (-1, 3) + t(3, 5)$ |

5. A normal to the plane containing the lines $(x, y, z) = (1, 2, 3) + s(2, -5, 1)$ and $(x, y, z) = (-2, 0, 1) + t(4, 0, -3)$ is

| A: (2, −7, 4) | B: (2, −5, 1) | C: (12, 6, 18) | D: (15, 10, 20) | E: (4, 0, −3) |

6. Which of the following is the equation of a line through the point $(1, 0, -3)$ and perpendicular to the plane $3x - 5y + z = 9$?

| A: $(x, y, z) = (3, -5, 1) + t(1, 0, -3)$ | B: $(x, y, z) = (1, 0, -3) + t(9, 3, -5)$ | C: $(x, y, z) = (1, 0, -3) + t(3, -5, 1)$ | D: $(x, y, z) = (9, 3, -5) + t(1, 0, -3)$ | E: none of A, B, C, or D |

7. Let $(a, b, c)$ be the point of intersection of the line $(x, y, z) = (1, 0, 2) + t(1, 2, 3)$ with the plane $x - y + 3z = 23$. Find $b$.

| A: 4 | B: 3 | C: −4 | D: −3 | E: 8 |

8. Find the value of $k$ for which $\mathbf{u} = (3, 2k, −1, −2)$ and $\mathbf{v} = (k, 2, −3, k)$ are orthogonal.

| A: $\frac{3}{5}$ | B: $\frac{1}{5}$ | C: $\frac{3}{5}$ | D: $\frac{1}{5}$ | E: 0 |

9. Give parametric equations of the line passing through the points $(-1, 3, 5)$ and $(2, 4, 6)$.

| A: \[
\begin{align*}
x &= \frac{1}{5} + 2t \\
y &= -\frac{3}{5} + 4t \\
z &= 5 + 6t
\end{align*}
| B: \[
\begin{align*}
x &= \frac{1}{5} + t \\
y &= -\frac{3}{5} + 7t \\
z &= 5 + t
\end{align*}
| C: \[
\begin{align*}
x &= \frac{2}{5} + t \\
y &= \frac{1}{5} + 3t \\
z &= 6 + 5t
\end{align*}
| D: \[
\begin{align*}
x &= \frac{1}{5} + t \\
y &= \frac{1}{5} + 2t \\
z &= 1 + 5t
\end{align*}
| E: \[
\begin{align*}
x &= \frac{1}{5} + t \\
y &= \frac{1}{5} + 2t \\
z &= 1 + 6t
\end{align*}

10. A point-normal form of the plane through the point $(2, 1, 8)$ with normal $(5, -4, 3)$ is

| A: $(5, -4, 3) \cdot ((x, y, z) - (2, 1, 8)) = 9$ | B: $(2, 1, 8) \cdot ((x, y, z) - (5, -4, 3)) = 9$ | C: $(5, -4, 3) \cdot ((x, y, z) - (2, 1, 8)) = 0$ | D: $(2, 1, 8) \cdot ((x, y, z) - (5, -4, 3)) = 0$ | E: none of A, B, C, or D |

11. Which of the following equations are linear in $x$, $y$, and $z$?

\[(i) \; x + y = z \]
\[(ii) \; xy = z \]
\[(iii) \; 2\sqrt{x} + 3y - z = 4 \]
(iv) \( \sqrt{2}x + 3y - z = 4 \)

\[ \begin{array}{cccc}
A: & (ii) & (iii) & \text{only} \\
B: & (i) & (iv) & \text{only} \\
C: & (i) & \text{only} \\
D: & \text{all of them} \\
E: & \text{none of them} \\
\end{array} \]

12. Which one of the following matrices is not in row-reduced echelon form?

(i) \( \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \)

(ii) \( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \)

(iii) \( \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \)

(iv) \( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \)

(v) \( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \)

\[ \begin{array}{cccc}
A: & (i) \\
B: & (ii) \\
C: & (iii) \\
D: & (iv) \\
E: & (v) \\
\end{array} \]

The following augmented matrix is to be used in questions 13, 14 and 15.

\[ \begin{bmatrix}
1 & 0 & 2 & 6 \\
0 & 1 & 1 & -3 \\
0 & 0 & 0 & a^2 - 16 \\
\end{bmatrix} \begin{bmatrix}
1 \\
-5 \\
\end{bmatrix} \]

13. The system of linear equations represented by the given augmented matrix has no solution when

\[ \begin{array}{cccc}
A: & a = 4 \\
B: & a \neq 4 \\
C: & a = -4 \\
D: & a \neq -4 \\
E: & a \neq \pm 4 \\
\end{array} \]

14. The system of linear equations represented by the given augmented matrix has a one-parameter family of solutions when

\[ \begin{array}{cccc}
A: & a = 4 \\
B: & a \neq 4 \\
C: & a = -4 \\
D: & a \neq -4 \\
E: & a \neq \pm 4 \\
\end{array} \]

15. The system of linear equations represented by the given augmented matrix has a two-parameter family of solutions when

\[ \begin{array}{cccc}
A: & a = 4 \\
B: & a \neq 4 \\
C: & a = -4 \\
D: & a \neq -4 \\
E: & a \neq \pm 4 \\
\end{array} \]

16. If \( A = \begin{bmatrix} 4 & -3 \\ 1 & 7 \end{bmatrix} \), \( B = \begin{bmatrix} 2 & 5 \\ -3 & -1 \end{bmatrix} \) and \( C = AB \), then the value of \( c_{21} \) is

\[ \begin{array}{cccc}
A: & 23 \\
B: & -19 \\
C: & -2 \\
D: & 17 \\
E: & -3 \\
\end{array} \]

17. Let \( A \) be a \( 3 \times 4 \) matrix and \( B \) be a \( 4 \times 3 \) matrix. Which one of the following is not defined?

\[ \begin{array}{cccc}
A: & A^T + B \\
B: & (A + B)^T \\
C: & AB \\
D: & (AA^T)^2 \\
E: & BA \\
\end{array} \]

18. If \( A = \begin{bmatrix} 2 & -2 \\ 8 & -7 \end{bmatrix} \) then \( A^{-1} = \)

\[ \begin{array}{cccc}
A: & \begin{bmatrix} -7 & 2 \\ -8 & 2 \end{bmatrix} \\
B: & \begin{bmatrix} -\frac{7}{2} & 1 \\ -4 & 1 \end{bmatrix} \\
C: & \begin{bmatrix} -7 & -2 \\ 8 & 2 \end{bmatrix} \\
D: & \begin{bmatrix} 2 & 2 \\ -8 & -7 \end{bmatrix} \\
E: & \begin{bmatrix} -\frac{7}{2} & 1 \\ -4 & 1 \end{bmatrix} \\
\end{array} \]
19. Let \( A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \). If \( A^{-1} = B = [b_{ij}] \), then the value of \( b_{12} \) is

| A: \(-2\) | B: \(-1\) | C: \(0\) | D: \(1\) | E: \(2\) |
---|---|---|---|---|

20. Let \( A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \) where \( A^{-1} = \begin{bmatrix} 1 & 2 & -4 \\ 3 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix} \). Consider the system of linear equations in \( x, y \) and \( z \):

\[
\begin{align*}
ax + by + cz &= 4 \\
ix + ey + fz &= 0 \\
gx + hy + iz &= 3
\end{align*}
\]

Find the value of \( y \).

| A: \(-4\) | B: \(-8\) | C: \(22\) | D: \(18\) | E: \(15\) |
---|---|---|---|---|

21. For what value of \( k \) is the rank of \( A = \begin{bmatrix} k & 1 \\ -1 & (k - 2) \end{bmatrix} \) equal to 1?

| A: \(-2\) | B: \(-1\) | C: \(0\) | D: \(1\) | E: \(2\) |
---|---|---|---|---|

22. The rank of

\[
\begin{bmatrix}
1 & -2 & 3 \\
2 & -4 & 6 \\
-1 & 2 & -3
\end{bmatrix}
\]
is

| A: \(0\) | B: \(1\) | C: \(2\) | D: \(3\) | E: \(4\) |
---|---|---|---|---|

23. Consider the following statements:

1. If \( A \) is any \( 5 \times 5 \) matrix with \( \text{rank}(A) = 4 \), then the linear system \( Ax = b \), with \( b \neq 0 \), must have infinitely many solutions.
2. If \( A \) is any \( 5 \times 5 \) matrix with \( \text{rank}(A) = 5 \), then \( A \) is invertible.
3. If \( A \) is any \( 5 \times 5 \) invertible matrix, then the linear system \( Ax = 0 \) has a unique solution.
4. Any homogeneous system of 5 linear equations in 7 unknowns always has a nontrivial solution.

Which one of the above statements is false?

| A: (1) | B: (2) | C: (3) | D: (4) |
---|---|---|---|

24. Let \( Ax = b \) be a system of 4 linear equations in 4 unknowns. Suppose \( \text{rank}(A) = 3 \) and \( \text{rank}[A \mid b] = 4 \). Then the system of equations has

<table>
<thead>
<tr>
<th>A: no solution</th>
<th>B: a unique solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>C: a one-parameter family of solutions</td>
<td>D: a two-parameter family of solutions</td>
</tr>
<tr>
<td>E: a three-parameter family of solutions</td>
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</tbody>
</table>

25. Consider a homogeneous system of six equations in five variables. Suppose the rank of the coefficient matrix is 2. Which one of the following is true?
26. Determine whether the following statement is true or false:
A homogeneous system with the same number of equations as unknowns always has a unique solution.

A: True    B: False

27. Determine whether the following statement is true or false:
If a linear system has no solution, the rank of the coefficient matrix must be less than the number of equations.

A: True    B: False

28. The 1,2-cofactor of
\[
\begin{pmatrix}
1 & 3 & 4 \\
0 & 5 & 7 \\
0 & 0 & 2 \\
\end{pmatrix}
\]
is
A: 0    B: 3    C: −3    D: −55    E: 55

29. \[\det\begin{bmatrix}
3 & 4 \\
2 & 5 \\
\end{bmatrix}\]

A: 3    B: 5    C: 7    D: 9    E: 11

30. \[\det\begin{bmatrix}
2 & 0 & 1 \\
−1 & 3 & −2 \\
−4 & 1 & −3 \\
\end{bmatrix}\]

A: 4    B: 2    C: 0    D: −2    E: −3

31. \[\det\begin{bmatrix}
1 & 5 & 1 & 8 \\
−2 & 7 & −2 & −2 \\
0 & 2 & 0 & 1 \\
3 & 6 & 3 & 0 \\
\end{bmatrix}\]

A: 0    B: 1    C: 2    D: 3    E: 4

32. \[\det\begin{bmatrix}
1 & 3 & 4 & 8 \\
0 & 5 & 7 & 3 \\
0 & 0 & 2 & 9 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}\]

A: −10    B: 10    C: −9    D: 9    E: 0

33. \[\det\begin{bmatrix}
0 & 0 & 0 & 2 \\
0 & 2 & 1 & 19 \\
0 & 0 & 5 & 21 \\
1 & 59 & 71 & 1 \\
\end{bmatrix}\]

A: The system has no solution.
B: The system has only the trivial solution.
C: The system has a two-parameter family of solutions.
D: The system has a three-parameter family of solutions.
E: The system has a four-parameter family of solutions.
5

A: 20 B: −20 C: 10 D: −10 E: 59

34. \( \text{det} \begin{bmatrix}
1 & 1 & 2 \\
1 & 2 & 3 \\
2 & 3 & 6 \\
3 & 3 & 8
\end{bmatrix} = \)

A: −4 B: −8 C: 96 D: 8 E: 4

Use the following for questions 35, 36 and 37:

\( A = \begin{bmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{bmatrix} \) and \( \text{det} A = 5 \)

35. Find \( \text{det} \begin{bmatrix}
a & g & d \\
b & h & e \\
c & i & f
\end{bmatrix} \).

A: 5 B: −5 C: \( \frac{1}{5} \) D: \( −\frac{1}{5} \) E: 0

36. Find \( \text{det} \begin{bmatrix}
g & h & i \\
3a & 3b & 3c \\
d & e & f
\end{bmatrix} \).

A: 0 B: 3 C: −3 D: 15 E: −15

37. Find \( \text{det} \begin{bmatrix}
d - a & e - b & f - c \\
3d & 3e & 3f \\
2g & 2h & 2i
\end{bmatrix} \).

A: 6 B: 30 C: −30 D: −6 E: \( \frac{5}{6} \)

38. If \( A \) is a 3 × 3 matrix and \( \text{det} A = 4 \), then \( \text{det}(-2A) \) is

A: −32 B: −16 C: 32 D: 16 E: −8

39. If \( A \) is a 4 × 4 matrix and \( \text{det} A = 3 \), then \( \text{det}(2A^{-1}) \) is

A: \( \frac{1}{3} \) B: 48 C: \( \frac{2}{3} \) D: 6 E: \( \frac{16}{3} \)

40. If \( A \) is a 3 × 3 matrix and \( \text{det} A = 4 \), then the rank of \( A \) is

A: 0 B: 1 C: 2 D: 3 E: 4

41. Find all values of \( c \) for which \( A \) is singular (has no inverse) if \( A = \begin{bmatrix}
c & c & 1 \\
-1 & c & c \\
2 & 1 & 1
\end{bmatrix} \).

A: \( c = -\frac{1}{2} \), 1 B: \( c = \pm \frac{1}{2} \) C: \( c = \pm \frac{1}{\sqrt{2}} \) D: \( c = \frac{1}{2} \), −1 E: \( c \neq \pm \frac{1}{\sqrt{2}} \)
42. If \( A \) is a \( 4 \times 4 \) matrix with \( \det A = 2 \), then \( \det(\text{Adj}(A)) \) is

<table>
<thead>
<tr>
<th></th>
<th>A: 2</th>
<th>B: ( \frac{1}{2} )</th>
<th>C: 8</th>
<th>D: (-2)</th>
<th>E: 16</th>
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</table>

In questions 43, 44, 45 and 46, determine whether the given statement is True or False.

43. If \( A \) and \( B \) are any \( n \times n \) matrices, then \( \det(A + B^T) = \det(A^T + B) \).

<table>
<thead>
<tr>
<th></th>
<th>A: True</th>
<th>B: False</th>
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</thead>
</table>

44. If \( A \) is a \( 5 \times 5 \) invertible matrix, then \( \det(-A) = \det A \).

<table>
<thead>
<tr>
<th></th>
<th>A: True</th>
<th>B: False</th>
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</table>

45. If \( A = A^{-1} \), then \( \det A = \pm 1 \).

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<th>B: False</th>
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46. If \( A \) is a \( 3 \times 3 \) matrix with \( A^T = -A \) then \( A \) is singular (has no inverse).

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<th>A: True</th>
<th>B: False</th>
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47. If \( \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = 3 \), \( \det \begin{bmatrix} a & e \\ c & f \end{bmatrix} = -4 \) and \( \det \begin{bmatrix} e & b \\ f & d \end{bmatrix} = 5 \), then the system of linear equations

\[
\begin{align*}
ax + by &= e \\
fx + dy &= f
\end{align*}
\]

has the unique solution

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<th>A: ( x = \frac{3}{5}, y = \frac{3}{4} )</th>
<th>B: ( x = -\frac{4}{3}, y = \frac{5}{3} )</th>
<th>C: ( x = \frac{4}{3}, y = -\frac{5}{3} )</th>
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</thead>
</table>

D: \( x = \frac{5}{3}, y = -\frac{4}{3} \) | E: \( x = -\frac{5}{3}, y = \frac{4}{3} \) |

48. If \( A = \begin{bmatrix} -1 & 2 & 4 \\ 2 & -5 & 3 \\ -3 & 7 & 5 \end{bmatrix} \), then the \( (2, 3) \) entry of \( \text{Adj} A \) is

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<th>C: 1</th>
<th>D: (-11)</th>
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49. If \( A \) is a \( 3 \times 3 \) matrix with \( \det A = 22 \) and \( \text{Adj} A = \begin{bmatrix} 10 & -4 & 2 \\ 2 & 8 & -4 \\ -7 & 5 & 3 \end{bmatrix} \), find the \( (1, 3) \) entry of \( A^{-1} \).

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<th>C: 2</th>
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50. If \( A = \begin{bmatrix} a & b & c \\ d & e & f \\ 3 & -1 & -1 \end{bmatrix} \) and \( \text{Adj} A = \begin{bmatrix} -2 & 4 & 10 \\ 7 & 1 & -5 \\ -13 & 11 & 5 \end{bmatrix} \), then \( \det A = \)

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<th>C: 30</th>
<th>D: 40</th>
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### INSTRUCTOR & COURSE EVALUATION

**THE UNIVERSITY OF WESTERN ONTARIO**

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<tr>
<th>Instructor Name</th>
<th>Teaching Faculty</th>
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<th>Subject</th>
<th>Course Number</th>
<th>Section</th>
<th>Year</th>
<th>Term</th>
<th>Enrolment</th>
<th>Responses</th>
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1.4.11 Mathematics 2120

Mathematics 2120A/B - Intermediate Linear Algebra I

A rigorous development of lines and planes in $\mathbb{R}^n$; linear transformations and abstract vector spaces. Determinants and an introduction to diagonalization and its applications including the characteristic polynomials, eigenvalues and eigenvectors.

Antirequisite(s): Mathematics 2211A/B and the former Mathematics 203b.

Prerequisite(s): Linear Algebra 1600A/B with a minimum mark of 60% or Mathematics 1120A/B with a minimum mark of 70% or permission of the Mathematics Department.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
Instructor.
Matthias Franz
Office: MC 134
E-mail: mfranz@uwo.ca

Class.
MWF 10:30–11:30 a.m., SSC 3018

Website.
http://www.math.uwo.ca/~mfranz/courses/2008_2120b.html

Textbook.

Prerequisites.
Linear Algebra 1600A/B (040a/b) with a minimum mark of 60% or Mathematics 1120A/B (060a/b) with a minimum mark of 70% or permission of the Mathematics Department.

Antirequisites.
Mathematics 2211B (283b) and the former Mathematics 203b.

Course Description.
Math 2120 is an intensive course on linear algebra, with the syllabus given below. If you have not done so already in Math 1120, you will learn to read and write mathematical proofs. Rigorous proofs are an essential feature of modern mathematics. Becoming familiar with them therefore is a prerequisite for success in this as well as in later courses.

Homework will be an important part of the course, so that you will have the feedback you need to develop new skills. Accordingly, Math 2120 is a demanding course (intellectually as well as in terms of time commitment), and you should be prepared to spend at least six hours per week on revision and homework. There will be approximately one assignment per week.
Syllabus.
Here is a brief outline of the topics covered in Math 2120. We will cover the majority of Chapters 1–5 of the book, in particular:
- Gaussian elimination,
- Vector spaces: dimension, span, bases,
- Linear transformations and change of bases,
- Determinants: their properties and how to compute them,
- Eigenvalues and eigenvectors; diagonalisation.

Evaluation of Student Performance.
Midterm Examinations. 2 midterms each worth 25% of the final grade.
Final Examination. 50% of the final grade.
Regarding Scholastic Offences and Penalties, see Section “Scholastic discipline for undergraduate students” of the Academic Calendar.

Make-up exams and conflicts.
Accommodation. All requests for accommodation for exams (midterms and final) must be channelled through the Dean’s office for approval. Examples would be requests to accommodate a religious holiday, medical issue or varsity sport event. See Section “Academic rights and responsibilities” in the Academic Calendar for details.

Conflicts. In the event that you must miss one of the midterms due to a valid conflict, the make-up exam will be given before the regular exam. Usually this will be earlier on the same day, or on the day before. If you are in this situation, you must let us know at least two weeks before the exam date.

See Section “Examination conflicts” of the Academic Calendar regarding conflicts involving final exams. If you have a conflict between the final exam and a midterm exam for a year-long course, you must contact the instructor of the other course.

In all cases, you should contact your instructor as soon as possible. Failure to follow these rules will result in a grade of zero.
1. [6 marks] Suppose $V$ and $W$ are vector spaces over a field $F$ and $T : V \to W$. Define the phrase, “$T$ is linear”.

2. [7 marks] Let

$$B = \left( \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)$$

be the standard ordered basis of $\mathbb{R}^{2 \times 3}$. Suppose that $T \in L(\mathbb{R}^{2 \times 3})$ satisfies

$$[T]_B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 0 & 0 \\ 1 & 2 & 3 & 4 & 5 & 0 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}.$$ 

Find

$$T \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}.$$ 

3. a) [6 marks] Let $F$ be a field and $A, B \in F^{n \times n}$. Define the phrase, “$A$ is similar to $B$”.

b) [6 marks] Let $F$ be a field and $A, B, C \in F^{n \times n}$. Suppose $A$ is similar to $B$ and $B$ is similar to $C$. Prove that $A$ is similar to $C$.

4. a) [6 marks] Suppose $V$ is a vector space over a field $F$ and $T \in L(V)$. Define the null space of $T$.

b) [6 marks] Let $T, U \in L(V)$. Let $W$ be the null space of $T$ and $Z$ be the null space of the composition $UT$. Prove that $W \subseteq Z$.

5. [7 marks] Let $G = \{ A \in \mathbb{R}^{n \times n} : \det A = 1 \text{ or } \det A = -1 \}$. Prove that if $A \in G$ then $A^{-1} \in G$. 


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6. Let \( A = \begin{pmatrix} -3 & 1 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \)

a) [7 marks] Find the characteristic polynomial and all eigenvalues of \( A \).

b) [7 marks] Find a basis for each eigenspace of \( A \).

7. Let \( V = \{ f(t) \in \mathbb{R}[t] : \deg(f(t)) < 5, f(1) = f(-1), f(2) = f(-2) \} \) and let \( T \in L(V) \) be defined by \( T(f(t)) = f''(t^2 + 1) \).

a) [7 marks] Prove that \( B = (1, t^2, t^4) \) is an ordered basis of \( V \).

b) [7 marks] Find \( [T]_B \).

c) [7 marks] Find the characteristic polynomial and all eigenvalues of \( T \).

8. [7 marks] Let \( V \) be a vector space and \( T \in L(V) \). A subspace \( W \) of \( V \) is called \( T \)-invariant provided \( T(\beta) \in W \) whenever \( \beta \in W \). Suppose \( T, U \in L(V) \) satisfy \( TU = UT \). Prove that the range of \( U \) is \( T \)-invariant.

9. [7 marks] An operator \( T \in L(V) \) is called idempotent provided \( T^2 = T \). Prove that if \( T \) is idempotent and \( \lambda \) is an eigenvalue of \( T \) then \( \lambda = 0 \) or \( \lambda = 1 \).

10. [7 marks] Fix an \( A \in \mathbb{R}^{n \times (n-1)} \) and note that for each \( X \in \mathbb{R}^{n \times 1} \) the matrix \( (A|X) \) is \( n \times n \). Define \( T : \mathbb{R}^{n \times 1} \to \mathbb{R} \) by \( T(X) = \det(A|X) \). Use the definition of determinant to show that \( T(X + Y) = T(X) + T(Y) \) for all \( X, Y \in \mathbb{R}^{n \times 1} \).
# Instructor & Course Evaluation

**The University of Western Ontario**

## Instructor Name: Franz, Matthias  
**Teaching Faculty:** Science  
**Teaching Department:** Mathematics  
**Subject:** Mathematics  
**Course Number:** 2120B  
**Section:** 001  
**Year:** 2008-09  
**Term:** Fall/Winter  
**Enrolment:** 18  
**Responses:** 11

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### Other Details

- **Percent of Classes Attended**
  - 90% or more: 9
  - 70% to 89%: 0
  - 50% to 69%: 1
  - 20% to 49%: 0

- **Expected Grade**
  - A: 3
  - B: 3
  - C: 3
  - D: 1

- **Course Status**
  - Required: 6
  - Optional: 4
  - Total: 10

- **Initial Level of Enthusiasm**
  - High: 5
  - Medium: 4
  - Low: 2
  - Total: 11

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Mathematics 2121

Mathematics 2121A/B - Intermediate Linear Algebra II

A continuation of the material of Mathematics 2120A/B including properties of complex numbers and the principal axis theorem; singular value decomposition; linear groups; similarity; Jordan canonical form; Cayley-Hamilton theorem; bilinear forms; Sylvester's theorem.

Antirequisite(s):

Prerequisite(s): Mathematics 2120A/B.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
Mathematics 2121B
Algebra of Boxes

INSTRUCTOR: Ján Mináč, MC 131


PREREQUISITES: Linear Algebra 1600A/B with a minimum mark of 60% or Mathematics 1120A/B with a minimum mark of 70% or permission of the Mathematics Department. Mathematics 2120A should be helpful. (If you did not yet take Mathematics 2120A, then please do speak with me.)

COURSE OUTLINE:

Like being hit by lightning during a thunderstorm is often the description given for the feeling of revelation and utter amazement that these seemingly complicated matrices can be treated just like “usual” numbers, except that multiplication is not necessarily commutative. But if they are like numbers then what will prime numbers be like, and how do you decompose vector spaces with linear operators into prime numbers? What will be special matrices such as the power of 2? The answers will be given by considering direct-sum decompositions, invariant direct sums, the primary decomposition theorem, rational and Jordan canonical forms, and invariant factors. What is the angle between two functions? When are two polynomials orthogonal to each other? Do these questions have sense, or are we just dreaming when we ask them? To answer these questions we shall consider inner product spaces and unitary operators. Wow! This should be just pure fun! We shall celebrate the marriage of algebra with geometry, with music, with cookies, and with great spirits. However even though we shall consider Jordan’s canonical forms, we may not necessarily play basketball.
EVALUATION OF STUDENT PERFORMANCE:

Will be discussed in class.

EXAMINATION DATES: The three-hour final examination will be given during the Final Examination period and will be scheduled by the Registrar's Office.

OFFICE HOURS: Mondays and Wednesdays: 12:00 p.m. – 12:45 p.m.
Or by appointment, or by other imaginative means.

STATEMENT ON ACADEMIC OFFENCES:

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation for Special Examination" form must be obtained from the Dean's Office immediately. For further information please see: http://www.uwo.ca/univsec/handbook/appeals/medical.pdf. A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services. The form can be found here: https://studentservices.uwo.ca/secure/medical_document.pdf. Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded and that all course prerequisites have been successfully completed. If the student does not have the prerequisites for a course, and does not have written permission from his or her Dean to enroll in the course, the student may be removed from the course and it will be deleted from the student's record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites. Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following web site: http://www.uwo.ca/univsec/handbook/appeals/scholoff.pdf
Time: 3 hours

This exam has 12 pages including the cover. Calculators are not permitted during the exam. Justify all answers and show all work. Throughout the exam, if not defined, $F$ is a field and $V$ is a non-zero $n$ dimensional $F$-vector space. If $V$ is a $n$ dimensional $F$ inner product space then unless otherwise stated, the field could be either $F = \mathbb{R}$ or $F = \mathbb{C}$.

1. Let $V$ be the vector space of functions $f : \mathbb{R}^2 \to \mathbb{R}$ of the form

$$f(x, y) = ax^2 + bxy + cy^2 + dx + ey + r$$

Then

$$\beta_0 = \{x^2, xy, y^2, x, y, 1\}$$

is an ordered basis of $V$. Let $T : V \to V$ be the linear mapping given by

$$T(f) = \frac{\partial}{\partial x} \int f(x, y) dy$$

Find

(a) The characteristic and minimal polynomials of $T$.

(b) The elementary divisors of $T$. (First find the dot diagram of each irreducible factor of $m_T$.)

(c) A Jordan basis for $T$ and a Jordan canonical form of $T$.

(d) A rational basis for $T$ and a rational canonical form of $T$.

2. Let $T : M_{22}(\mathbb{C}) \to M_{22}(\mathbb{C})$ be given by

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ib & ia \\ d & -c \end{bmatrix}$$

Assume that $M_{22}(\mathbb{C})$ has the Frobenius inner product.

(a) Find $T^* : M_{22}(\mathbb{C}) \to M_{22}(\mathbb{C})$. (Find the matrix of $T^*$ with respect to an orthonormal basis first! Then determine $T^*\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$.)
(b) Is $T$ normal, unitary, skew-adjoint or self-adjoint? Why or why not?

(c) What are the eigenvalues of $T$ and their algebraic and geometric multiplicities?

(d) Find an orthonormal basis for each eigenspace.

(e) Orthogonally diagonalise $A = [T]_{\beta_0}$. (Use (d)).

3. Let $\beta = \{1 + x + x^2, 1 + x, 1\}$ be a basis for $P_2(\mathbb{R})$. Let $\beta_0 = \{1, x, x^2\}$ be the standard basis for $P_2(\mathbb{R})$. Let $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be the map $T(f(x)) = (x - 1)f'(x)$.

(a) Find the dual basis $\beta^* = \{f_1, f_2, f_3\}$ of $\beta$. For each element of the dual basis, find $f_i(a + bx + cx^2)$, $i = 1, 2, 3$.

(b) For the subspace $W = \text{Span}\{1 + x + x^2\}$, find a basis of $W^0$.

(c) Find $[T^T]_{\beta_0^*}$.

(d) Find a basis for $N(T^T)$. (First find a basis of $N([T^T]_{\beta_0^*})$). For each element $g_i$ of the basis of $N(T^T)$, find $g_i(a + bx + cx^2)$.

4. (a) Suppose $A \in M_{nn}(\mathbb{R})$ has $m_A(x) = (x - 1)^2(x - 2)^2(x - 3)$. Suppose $\det(A) = 72$, $\text{tr}(A) = 16$ and $\dim(N(A - I_n)) = 2$. Find the characteristic polynomial of $A$, the dot diagram for each irreducible factor of $m_A$, the elementary divisors of $A$, and the Jordan form of $A$.

(b) Suppose $T : V \to V$ is a real linear operator with $c_T(x) = (x^2 + 1)^{10}$.

Suppose $p(x) = x^2 + 1$, $\text{rank}(p(T)) = 10$, $\text{rank}(p^2(T)) = 4$ and $p^3(T) = 0$. What are the minimal polynomial, the dot diagram of $T$ with respect to $p$, the elementary divisors, and the RCF of $T$ over $\mathbb{R}$.

5. Recall that the nilpotent index of a nilpotent matrix $N$ is the smallest positive integer $k$ such that $N^k = 0$.

Show that 2 nilpotent 6 by 6 matrices with nilpotent index 3 and rank 4 must be similar. Show by example that there exist 2 nilpotent 7 by 7 matrices with nilpotent index 3 and rank 4 that are not similar. [Hint: Consider dot diagrams and Jordan forms]
6. Suppose $T = ip_1 - 2ip_2 + 3ip_3$ is a spectral decomposition of $T : V \to V$ where $V$ is a $\mathbb{C}$-inner product space. What are the eigenvalues of $T$? Is $T$ self-adjoint, skew-adjoint or unitary? Why or why not? What is $m_T$? If $\dim(R(p_1)) = 2$, $\dim(R(p_2)) = 4$ and $\dim(R(p_3)) = 3$, what are $c_T$ and $\dim(V)$?

7. Let $T : M_{nn}(\mathbb{R}) \to M_{nn}(\mathbb{R})$ be defined by $T(A) = A^T$. Give $M_{nn}(\mathbb{R})$ the Frobenius inner product $\langle A, B \rangle = \text{tr}(B^T A)$.

(a) What is $T^2$ and $T^{-1}$?
(b) What is $m_T$ and the eigenvalues of $T$. (Use (a) and justify your answer).
(c) Show that $\langle T(A), B \rangle = \langle A, T(B) \rangle$ for all $A, B \in M_{nn}(\mathbb{R})$.
(d) What is $T^*$? (Use (c) and no extra work). Is $T$ normal, self-adjoint, skew-adjoint or orthogonal? Why or why not?
(e) Why is $T$ orthogonally diagonalisable? Identify the eigenspaces of $T$ with (known) subspaces of matrices. Write down the orthogonal decomposition of $M_{nn}(\mathbb{R})$ into $T$ eigenspaces in terms of this identification.

8. Let $V$ be a real inner product space.

(a) Show that $T : V \to V$ is orthogonal if and only if $\langle T(x), T(y) \rangle = \langle x, y \rangle$ for all $x, y \in V$.
(b) Show that if $T : V \to V$ is orthogonal, then $\| T(x) \| = \| x \|$ for all $x \in V$.
(c) What is the formula for $\cos(\theta)$ if $\theta$ is the angle between $x, y \in V$ (in terms of the inner product).
(d) Conclude from (a),(b),(c) that orthogonal maps preserve lengths and angles.
### Instructor & Course Evaluation

**The University of Western Ontario**

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<th>Teaching Department</th>
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**Percent of Classes Attended**

- 90% or more: 2
- 70% to 89%: 4
- 50% to 69%: 0
- 20% to 49%: 0

**Expected Grade**

- A: 1
- B: 5
- C: 0
- D: 0

**Course Status**

- Required: 4
- Optional: 2
- Total: 6

**Initial Level of Enthusiasm**

- High: 2
- Medium: 3
- Low: 1
- Total: 6

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1.4.13 Mathematics 2122

Mathematics 2122A/B - Real Analysis I

A rigorous introduction to analysis on the real line, primarily for honors students. Sets, functions, natural numbers, Axioms for the real numbers, Completeness and its consequences, Sequences and limits, Continuous and differentiable functions, The Mean Value Theorem.

Antirequisite(s):
Prerequisite(s): Calculus 1501A/B or Applied Mathematics 1413, with a minimum mark of 60%, or Calculus 1301A/B with a minimum mark of 85%.

Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 4 lecture hours, 0.5 course.
Math 2122, Fall 2008
Real Variables I
Syllabus

Instructors:
○ Section 001: Rasul Shafikov, MC 112, shafikov@uwo.ca (emails will be answered within 48 hours).
○ Section 002: Janusz Adamus, MC 116, jadamus@uwo.ca


Course web page: http://www.math.uwo.ca/~shafikov/2122/
Visit this page for up to date information on the course.

Course Description:
○ Section 001: TuTh 9:30 - 11:30 UC 212.
○ Section 002: TuTh 11:30 - 1:30 UC 212.

This course is intended primarily for honours students and provides a rigorous introduction to the analysis of real-valued functions of a real variable. Topics will include: real numbers - axioms and properties; sets - cardinality, Cantor's theorem; sequences and series - limit and convergence; continuous and differentiable functions. This approximately corresponds to Chapters 1 and 2, Sections 3.2 and 3.3, Chapter 4, and the beginning of Chapter 5 of the textbook.

The distinguished feature of this course is its emphasis on theory and proof.

Prerequisites: Calculus 1501a/b or Applied Mathematics 1413, with a minimum mark of 60%, or Calculus 1301a/b with a minimum mark of 85%. The following statement is required by Senate regulations:

"Unless you have either the requisites for this course or written special permission from your Dean to enrol in it, you may be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisites."

What is expected of a student? The goal of Mathematics 2122a and 2123b is a good understanding of the fundamental concepts and methods of mathematical analysis, as well as a facility in reading and writing proofs. Key expectations:

1. Class attendance is required, class participation is expected. The latter includes both asking questions and responding to instructor's questions.
2. Preparation for each class will involve reading your class notes and the textbook. This should be done with paper and pencil at hand, both for recording any comments or questions that may occur to you and for checking any claims that are made in proofs and examples.
3. Your success in this course will depend in great measure on your doing all the assigned (and unassigned too!) exercises and keeping up-to-date.

Homework: Homework will be assigned biweekly, due before class on Tuesday.
1. (a) Give an example of a function \( f : \mathbb{R} \rightarrow \mathbb{R} \) such that \( f(K) \) is compact for every compact set \( K \), but \( f \) is not a continuous function on \( \mathbb{R} \).

(b) Give an example of a continuous function \( f : \mathbb{R} \rightarrow \mathbb{R} \) such that \( f^{-1}(K) \) is not compact for some compact \( K \).

2. (a) Let \( f : \mathbb{R} \rightarrow \mathbb{R} \). Give a precise definition that \( f(x) \) converges to a finite number \( L \), as \( x \rightarrow \infty \).

(b) Recall that a function \( f(x) \) is called increasing if \( f(x_1) \leq f(x_2) \) for \( x_1 \leq x_2 \). Prove that a bounded increasing function \( f : \mathbb{R} \rightarrow \mathbb{R} \) converges to some finite \( L \) as \( x \rightarrow \infty \).

3. Let \( S \) and \( T \) be compact subsets of \( \mathbb{R} \). Define a new set

\[ S + T = \{ s + t : s \in S \text{ and } t \in T \} \]

Prove that \( S + T \) is also compact.

4. Let \( (x_n) \) be an arbitrary sequence of real numbers. Prove that the set \( B \) of limit points of the set \( A = \{ x_n : n \geq 1 \} \) is closed.

5. Let \( f(x) \) be a continuous function on \([0, 1] \), and let \( g(x) = x^2 \). Prove that the function \( g \circ f(x) \) is uniformly continuous on \([0, 1] \).

6. (a) State Cauchy criterion for convergence of a series \( \sum a_n \).

(b) Let \( \sum a_n, \sum b_n \) be two series, with \( a_n > 0 \) for all \( n \). Prove the Limit Comparison Test for series: if \( \sum a_n \) converges, and

\[ \lim_{n \to \infty} \frac{b_n}{a_n} = L, \]

where \( L \) is some real number, then \( \sum b_n \) converges.
## INSTRUCTOR & COURSE EVALUATION
### THE UNIVERSITY OF WESTERN ONTARIO

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### Questions and Results

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### Additional Data

- **Percent of Classes Attended**
  - 90% or more: 8
  - 70% to 89%: 2
  - 50% to 69%: 0
  - 20% to 49%: 0

- **Expected Grade**
  - A: 2
  - B: 3
  - C: 4
  - D: 0

- **Course Status**
  - Required: 6
  - Optional: 4
  - Total: 10

- **Initial Level of Enthusiasm**
  - High: 3
  - Medium: 8
  - Low: 0
  - Total: 11
1.4.14 Mathematics 2123

**Mathematics 2123A/B - Real Analysis II**


**Antirequisite(s):** The former Mathematics 306a/b.

**Prerequisite(s):** Mathematics 2122A/B with a minimum mark of 60%, or permission of the Department.

**Corequisite(s):**

**Pre-or Corequisite(s):**

**Extra Information:** 3 lecture hours, 0.5 course.
Math 2123B, Winter 2009
Real Variables II
Syllabus

Instructor: Janusz Adamus, MC 116, jadamus@uwo.ca.

The same book as used in Mathematics 2122a.

Course web site: http://www.math.uwo.ca/~jadamus/08-09/2123W2009/.
Visit this site regularly for homework assignments and other up to date information on the course.

Course Description: MWF 9:30 - 10:30 PB36.
This course is the continuation of 2122a, and is intended primarily for honours students. The course
provides a rigorous introduction to the analysis of real-valued functions of a real variable. Topics will
include: basic topology of the real line, functional sequences and series, Riemann integral.
The distinguished feature of this course is its emphasis on theory and proof.

Prerequisites: Real Variables I, Math2122 (former Math207) with a minimum mark of 60%, or by
permission of the Department of Mathematics. The following statement is required by Senate regula-
tions:

“Unless you have either the requisites for this course or written special permission from your Dean to
enrol in it, you may be removed from this course and it will be deleted from your record. This decision
may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from
a course for failing to have the necessary prerequisites.”

What is expected of a student? The goal of Mathematics 2122a and 2123b is a good understanding
of the fundamental concepts and methods of mathematical analysis, as well as a facility in reading and
writing proofs. Key expectations:

(1) Class attendance is required, class participation is expected. The latter includes both asking
questions and responding to instructor’s questions.

(2) Preparation for each class will involve reading your class notes and the textbook. This should
be done with paper and pencil at hand, both for recording any comments or questions that may
occur to you and for checking any claims that are made in proofs and examples.

(3) Your success in this course will depend in great measure on your doing all the assigned (and
unassigned too!) exercises and keeping up-to-date.

Homework: Homework will be assigned weekly, due before class on Friday.

(1) The work that is turned in for marking must be written up neatly and legibly with attention
to the requirement of grammar and style that make writing easy to read and understand. This
will generally involve, at the very least, one rough draft followed by a careful rewrite.

(2) Discussion of the assignments with classmates will be permitted but no collaboration of any kind
on the write-ups will be tolerated. Students who allow copying of their work, as well as those
who do the copying, will get zero for the assignment in question. In addition, other penalties
may be imposed in keeping with the guidelines set out by Senate Regulations on Scholastic
Offences.
1. (a) Give a precise statement of the Mean Value Theorem.
(b) Prove or disprove the following statement:
   \( f : \mathbb{R} \to \mathbb{R} \) is a differentiable contraction, then \( f' \) is a bounded function.

2. Find the radius of convergence \( R \) of the power series
   \[
   \sum_{n=1}^{\infty} \frac{5^n + (-3)^n}{n} x^n,
   \]
   and discuss convergence at the endpoints \( x' = -R \) and \( x'' = R \).

3. Prove that the function \( f(x) = \sum_{n=2}^{\infty} \frac{1}{x^2 + n^2} \) is differentiable on \( \mathbb{R} \), and the derivative function \( f' \) is continuous on \( \mathbb{R} \).

4. Find the domain of the function \( f(x) = \sum_{n=2}^{\infty} \frac{n}{x^2 + n^2} \). (I.e., determine for which values of \( x \) the given power series converges).

5. Let \( \{r_n\}_{n=1}^{\infty} \) be a list of all the rational numbers (without repetitions). Prove or disprove the following statement:
   If a function \( f : [0,1] \to \mathbb{R} \) satisfies \( f(r_k) = \ln(k) \) for all \( r_k \in [0,1] \), then \( f \) is not Riemann-integrable.

6. Prove or disprove the following statement:
   If \( D_f \) is the set of discontinuities of a Riemann-integrable function \( f : [a,b] \to \mathbb{R} \), and a function \( g : [a,b] \to \mathbb{R} \) satisfies \( g(x) = 0 \) for all \( x \in [a,b] \setminus D_f \), then \( g \) is Riemann-integrable.

**Bonus.** Prove that the function \( f(x) = \frac{1}{x - 2} \) is analytic in \( \mathbb{R} \setminus \{2\} \); that is, show that, for every point \( x_0 \neq 2 \), there is a positive \( \delta \) and a sequence of real numbers \( \{c_n\}_{n=0}^{\infty} \), such that
   \[
   f(x) = \sum_{n=0}^{\infty} c_n (x - x_0)^n,
   \]
   for all \( x \in (x_0 - \delta, x_0 + \delta) \).
   Given \( x_0 \neq 2 \), what is the greatest possible \( \delta \) for which the above equality holds?
The table below provides a detailed analysis of instructor and course evaluation metrics for the University of Western Ontario.

### Questionnaire Responses

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### Percentage of Classes Attended

- 90% or more: 5
- 70% to 89%: 1
- 50% to 69%: 0
- 20% to 49%: 0

### Expected Grade

- A: 3
- B: 1
- C: 2
- D: 0

### Course Status

- Required: 5
- Optional: 1
- Total: 6

### Initial Level of Enthusiasm

- High: 0
- Medium: 4
- Low: 2
- Total: 6

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Mathematics 2124A/B - Introduction to Mathematical Problems

Primarily for Mathematics students, but will interest other students with ability in and curiosity about mathematics in the modern world as well as in the past. Stresses development of students' abilities to solve problems and construct proofs. Topics will be selected from: counting, recurrence, induction; number theory; graph theory; parity, symmetry; geometry.

Antirequisite(s):

Prerequisite(s): Calculus 1501A/B or Applied Mathematics 1413, with a minimum mark of 60%, or Calculus 1301A/B with a minimum mark of 85% or permission of the instructor.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 2124B

2009-2010

INSTRUCTOR: Gord Sinnamon


PREREQUISITES: Calculus 1501A/B or Applied Mathematics 1413, with a minimum mark of 60%, or Calculus 1301A/B, with a minimum mark of 85% or permission of the instructor.

COURSE OUTLINE: Primarily for Mathematics students, but will interest other students with ability in and curiosity about mathematics. Stresses development of students' abilities to solve problems and construct proofs. Topics will be selected from: counting, recurrence, induction; number theory; graph theory; parity, symmetry; geometry.

WHAT IS EXPECTED OF THE STUDENT: The course involves not only learning new mathematical techniques and tools, but also requires each student to engage in creative application of mathematical ideas to solve a wide range of mathematical problems. The course emphasizes clear and effective communication of problems and solutions. Proficiency in English is required and will be taken into account in the grading of tests and assignments.

Students are expected to attend all lectures and complete all quizzes, assignments and examinations. No aids are permitted in quizzes and examinations.

EVALUATION OF STUDENT PERFORMANCE:

1. In-Class Quizzes: 30% (Dates to be announced.)
2. Written Assignments: 30%
3. Final examination: 40%

EXAMINATION DATES:

The three-hour final examination will be given during the Final Examination period and will be scheduled by the Registrar's Office.

The Fine Print: If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation for Special Examination" form must be obtained from the Dean's Office immediately. For further information please see: http://www.uwo.ca/univsec/handbook/appeals/medical.pdf. A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services. The form can be found here: https://studentservices.uwo.ca/secure/medical_document.pdf. Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded and that all course prerequisites have
been successfully completed. If the student does not have the requisites for a course, and does not have written special permission from his or her Dean to enroll in the course, the student may be removed from the course and it will be deleted from the student’s record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites. Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site: http://www.uwo.ca/univsec/handbook/appeals/scholoff.pdf

Questions or comments about this page?
Math 2124B Final Examination
April 24, 2009, 2:00 p.m.–5:00 p.m.

All solutions should be well-written and complete. A poorly written complete solution will not receive full credit. However, a well-written partial solution may receive substantial credit.

IMPORTANT: A complete paper consists of solutions to seven of the following eleven questions including at least three of the last five (B7, B8, B9, B10, B11). Clearly indicate which questions you wish to have considered.

A1. [15 marks] Draw two circles in the plane with the property that no line is tangent to both of them. No proof is required.

A2. [15 marks] Find an integer \(x\), greater than 1000, such that \(4x \equiv 7 \mod{19}\) and \(x - 3 \equiv 5 \mod{20}\).

A3. [15 marks] A Math 2124B student recently found a pile of old Hitchhiker’s Guide to the Galaxy Addition Flash Cards in a bottom drawer. The cards looked like, for example,

\[
\begin{array}{|c|}
\hline
10 \\
+10 \\
\hline
\end{array}
\quad
\begin{array}{|c|}
\hline
13 \\
+27 \\
\hline
\end{array}
\quad
\begin{array}{|c|}
\hline
29 \\
+21 \\
\hline
\end{array}
\quad
\begin{array}{|c|}
\hline
30 \\
+30 \\
\hline
\end{array}
\]

Originally, the deck consisted of all possible cards with the top and bottom numbers between 10 and 30, inclusive. Now, however, many of the cards have been lost. The student counted the cards in the pile without looking at them and immediately announced, “There are at least two cards here with the same sum.” What is the minimum number of cards there could be in the pile?

A4. [15 marks] Let \(F\) be a finite set of non-zero rational numbers and define

\[
s(F) = \sum_{r \in F} \left( r - \frac{1}{r} \right).
\]

For example, if \(F = \{1/5, 3, -2/3\}\) then \(s(F) = (1/5 - 5) + (3 - 1/3) + (-2/3 + 3/2) = -13/10\).

Show that for any such set \(F\) there is a set \(G\) such that \(s(G) = -s(F)\).

A5. [15 marks] Let \(a_1 = 1\) and \(a_{n+1} = \left(1 + \frac{1}{2n}\right) a_n\) for \(n = 1, 2, \ldots\). Prove that

\[
a_n = \left(\frac{2n}{n}\right)^{2n
\frac{2n}{2n}}
\]

for \(n = 1, 2, \ldots\).
A6. [15 marks] Find the positive integer $n$ such that, if $r_0 = 5 + 20n$ and $r_1 = 6 + 5n$ then the Euclidean Algorithm applied to $r_0$ and $r_1$ begins,

\[
\begin{align*}
  r_0 &= 3r_1 + r_2 \\
  r_1 &= 3r_2 + r_3 \\
  r_2 &= 1r_3 + r_4 \\
  r_3 &= \ldots
\end{align*}
\]

B7. [15 marks] For a certain cargo ship to be properly trimmed, the difference in weight between the cargo in the forward hold and the cargo in the aft hold must not exceed 40 tonnes. The ship is to carry three containers weighing 40, 60, and 90 tonnes, respectively, as well as a large container of unknown weight. The large container is known to weigh less than 230 tonnes but will be accurately weighed before any cargo is loaded. Show that no matter what the weight of the large container is, it will be possible to load the four containers into the two holds so that the ship will be properly trimmed.

B8. [15 marks] A certain pond at Ace Sport Fishing is stocked with $n$ different species of fish, but the number of fish of each species is unknown. It is known that every fish is equally likely to be caught, whatever its species. Ace has a strict “catch and release” policy; once a fish is caught it is immediately returned to the pond and may be caught again. Show that if I catch two fish from that pond, the probability that they are both of the same species is at least $1/n$.

B9. [15 marks] Three siblings, Anne, Branwell, and Charlotte, were planning to join a book club. Each one made a list of the books in the club’s catalogue that he or she had already read. It turned out that each of these three lists had the same number of books on it. Also, Anne’s and Branwell’s lists had 17 books in common, Anne’s and Charlotte’s lists had 8 books in common, and Branwell’s and Charlotte’s lists had 9 books in common. Only 3 books from the catalogue were on all three lists. Eventually, they decided not to join the club because only 15 of the 149 books in the catalogue were not on any of the three lists. How many books were on Charlotte’s list?

B10. [15 marks] A process begins with three positive integers. Every day, at midnight, each of the three numbers is replaced by the difference (larger minus smaller) of the two other numbers. Prove that eventually one of the numbers will become zero.

B11. [15 marks] Each year on St. Patrick’s Day a magic spell replaces every gold coin in Carrie Leprechaun’s magic pot by 17 gold coins. The very next day Carrie buys as many pairs of shoes as she can with the coins in her pot. Shoes cost 37 gold coins per pair. The remaining coins stay in her pot until next year. Carrie started with 2 gold coins on the day before St. Patrick’s day in 1900. How many coins were in the pot on the day before St. Patrick’s day in 2009?
<table>
<thead>
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<th>Instructor Name: Sinnamon, Gordon J</th>
<th>Teaching Faculty: Science</th>
<th>Teaching Department: Mathematics</th>
<th>Subject: Mathematics</th>
<th>Course Number: 2124B</th>
<th>Section: 001</th>
<th>Year: 2008-09</th>
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Mathematics 2155A/B - Discrete Structures I

This course provides an introduction to logical reasoning and proofs. Topics include sets, counting (permutations and combinations), mathematical induction, relations and functions, partial order relations, equivalence relations, groups and applications to error-correcting codes.

Antirequisite(s): Software Engineering 2251A/B

Prerequisite(s): 1.0 course from: Mathematics 1120A/B, Applied Mathematics 1413, Calculus 1000A/B or 1100A/B, Calculus 1301A/B or 1501A/B, Linear Algebra 1600A/B, or the former Mathematics 030 (in each case with a minimum mark of 60%).

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 4 lecture hours, 0.5 course.
Mathematics 2155a 2009

Course Outline

Instructor: S. Rankin

Textbook:


Prerequisites:

- One full course or equivalent chosen from Mathematics 1120A/B, Applied Mathematics 1413, Calculus 1000A/B or 1100A/B, Calculus 1301A/B or 1501A/B, Linear Algebra 1600A/B, or the former Mathematics 030 (in each case with a minimum mark of 60%).

Course Outline:

- Mathematical induction.
- Set theory.
- Counting: permutations, combinations, the binomial theorem.
- Relations and functions.
- Equivalence relations and partitions.
- Partial orders and lattices.
- Group theory and applications to error detecting and correcting codes.

Evaluation of Student Performance:

- Term mark of 10%, determined by assignments.
- Two 2.5 hour non-cumulative term tests, each worth 20% of the final mark.
- A three hour cumulative final examination, worth 50% of the final mark.

Examination Dates:

- First Midterm: Saturday, October 17, 2009, 9:30--noon, in TC, rooms 341, 342, and 343.
- Final examination in the December exam period, to be scheduled by the Office of the Registrar (the exam period is Dec. 11 - Dec. 22). Please do not make travel plans until you have obtained your final exam schedule.

Statement on Academic Offences: Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following web site: http://www.uwo.ca/univsec/handbook/appeals/scholoff.pdf.
Medical Accommodation

- For UWO Policy on Accommodation for Medical Illness see:
- Downloadable Student Medical Certificate (SMC): [https://studentservices.uwo.ca](https://studentservices.uwo.ca) under the Medical Documentation heading
- Students seeking academic accommodation on medical grounds for any missed tests, exams, participation components and/or assignments worth 10% or more of their final grade must apply to the Academic Counselling office of their home Faculty and provide documentation. Academic accommodation cannot be granted by the instructor or department.
Mathematics 2155a Final Exam December 16, 2009

Instructions: Print your name on the SCANTRON answer sheet. Sign the SCANTRON answer sheet, and use a PENCIL to mark your answers to questions 1–25 on the SCANTRON answer sheet. Use a PENCIL to record your student number on the question sheet.

A1. [2 marks] In how many ways can 6 couples be seated at a circular table if the two members of each couple must sit side by side?

A: $6!2^6$  B: $5!2^5$  C: $5!$  D: $5!6^2$  E: $5!2^6$

A2. [2 marks] For any sets $A$ and $B$, $A \cap (A \Delta B)$ is equal to:

A: $A$  B: $B$  C: $A \cup B$  D: $A \cap B$  E: $A - B$

A3. [2 marks] How many surjective functions $f$ are there from $J_4$ to $J_5$ with the property that $f(1) = 3$?

A: 36  B: 6  C: 30  D: 12  E: $4! + \binom{4}{3}3!$

A4. [2 marks] Recall that $\mathbb{Z}^+$ denotes the positive integers, and that for any real number $x$, $|x|$ is the greatest integer less than or equal to $x$. Which of the following functions $f : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ are surjective?

(i) $f(n) = \lfloor (n + 1)/2 \rfloor$
(ii) $f(n) = n + 1$
(iii) $f(n) = \binom{n+1}{2}$

A: (i)  B: (ii)  C: (i), (ii)  D: (iii)  E: (ii), (iii)

A5. [2 marks] Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be partially ordered by the division relation (that is, for $a, b \in A$, we say that $a \leq b$ if $a$ is a divisor of $b$). How many maximal elements are there for this partial order relation?

A: 1  B: 2  C: 3  D: 4  E: 5

A6. [2 marks] How many relations on $J_4$ are reflexive and symmetric, and contain the pair $(3, 4)$?

A: $2^{10}$  B: $2^{12}$  C: $4!$  D: $2^6$  E: $2^5$

A7. [2 marks] The number of binary operations on $J_4$ that have 2 as an identity element is:

A: $2^9$  B: $3^9$  C: 1  D: $3^1$  E: $3^3$

A8. [2 marks] Consider the relations $R_1$, $R_2$ and $R_3$ on $\mathbb{Z}^+$ (the positive integers) defined by

$(a, b) \in R_1 \iff a^2 - a = b^2 - b$; 
$(a, b) \in R_2 \iff |a - b| < 4$; 
$(a, b) \in R_3 \iff a + b$ is odd.

Exactly which of the relations $R_1$, $R_2$, $R_3$ are equivalence relations?

A: $R_1$, $R_3$  B: $R_3$  C: $R_1$, $R_2$  D: $R_1$  E: None of them

A9. [2 marks] Relations $R_1$, $R_2$, $R_3$ on $\mathbb{Z}^+$ are given by:

$(a, b) \in R_1 \text{ if } a \text{ is a divisor of } b$; 
$(a, b) \in R_2 \text{ if } a + b \text{ is even}$; 
$(a, b) \in R_3 \text{ if } |a - b| \geq 3$.

Exactly which of the relations $R_1$, $R_2$, $R_3$ are partial order relations on $\mathbb{Z}^+$?

A: $R_2$  B: $R_1$, $R_3$  C: $R_1$, $R_2$  D: $R_3$  E: None of A, B, C, D
A10. [2 marks] Let * denote the binary operation defined on \( \mathbb{Z} \) by \( x * y = x + y + 4 \) for all \( x, y \in \mathbb{Z} \). Exactly which of the following properties does * have?

(i) It is commutative.
(ii) It is associative.
(iii) It has an identity element.

A: (i), (ii)  B: (i), (iii)  C: (iii)  D: (ii), (iii)  E: All of them

A11. [2 marks] The binary operation * on \( \mathbb{N} \) given by \( m * n = |m - n| \) (the absolute value of the difference) is:

A: commutative, associative, and has an identity
B: associative but has no identity and is not commutative
C: commutative and has an identity, but is not associative
D: commutative but not associative and has no identity
E: None of A, B, C, D

A12. [2 marks] If \( G \) is a group of size 15 and \( H \) is a subgroup of \( G \) of size at least 6, what is the size of \( H \)?

A: 6  B: 9  C: 12  D: 15  E: None of A, B, C, D

A13. [2 marks] Suppose that \( G \) is a group with 120 elements and \( H \) is a subgroup of \( G \) of size 15. The number of left cosets of \( H \) in \( G \) is:

A: 24  B: 18  C: 12  D: 6  E: None of A, B, C, D

A14. [2 marks] For any sets \( A \) and \( B \), \( (A \triangle B) \triangle (A \cap B) \) is equal to:

A: \( A \)  B: \( B \)  C: \( A \cap B \)  D: \( A \cup B \)  E: None of A, B, C, D

A15. [2 marks] How many binary sequences of length 8 begin with 11 and do not end with 111?

A: 56  B: 86  C: 26  D: 62  E: None of A, B, C, D

A16. [2 marks] In how many ways can the letters of the word REARRANGEMENT be arranged in a row if the letter T must appear before any R’s appear? (For example, ATEENR-ERRAGMN is such an arrangement).

A: \( \binom{13}{1}\binom{12}{3} \)  B: \( \binom{13}{4} \)  C: \( 13!/4!3!2!2! \)  D: \( 13!/3! \)  E: None of A, B, C, D

A17. [2 marks] How many equivalence relations are there on the set \( J_5 \)?

A: 15  B: 52  C: 16  D: \( 2^43^6 \)  E: \( 2^{12} \)

A18. [2 marks] The number of surjective functions from \( J_5 \) to \( J_4 \) is:

A: 360  B: 480  C: 240  D: 120  E: None of A, B, C, D

A19. [2 marks] How many left inverses are there for the function \( f = \binom{123}{3,1,5} \) from \( J_3 \) to \( J_5 \)?

A: 1  B: 0  C: 27  D: 9  E: None of A, B, C, D

A20. [2 marks] Given that the relation

\[ R = \{ (1, 1), (1, 3), (2, 2), (2, 3), (2, 4), (3, 3), (4, 4), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5) \} \]

is a partial order relation on the set \( J_5 \), which of the following diagrams is the Hasse diagram of \( R \)?
A21. [2 marks] Exactly which of the following are groups?

(i) The set of all negative real numbers, with operation multiplication.
(ii) The set of all positive real numbers, with operation division (that is, the binary operation \( \ast \) is given by \( x \ast y = x/y \) for all positive reals \( x \) and \( y \)).
(iii) The set of all non-zero real numbers, with operation addition.

A: (i), (ii) B: (i), (iii) C: (ii), (iii) D: (ii) E: None of A, B, C, D

A22. [2 marks] The order of \( \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \) \( \in S_5 \) is:

A: 2 B: 3 C: 4 D: 5 E: 6

A23. [2 marks] The inverse of \( \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 5 \end{pmatrix} \) \( \in S_5 \) is:

A: \( \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 1 & 5 \end{pmatrix} \) B: \( \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 4 \\ 4 & 5 & 2 \end{pmatrix} \) C: \( \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 4 \\ 4 & 5 & 2 \end{pmatrix} \) D: \( \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 4 & 5 & 1 \end{pmatrix} \) E: \( \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 2 \\ 4 & 1 & 5 \end{pmatrix} \)

A24. [2 marks] Which of the following statements are true for every positive integer \( n \)?

(i) there is an abelian group of order \( n \);
(ii) in every group of order \( n \), there is some element with order \( n \);
(iii) if \( G \) is a finite group and \( g \in G \) has order \( n \), then \( g^{-1} = g^{n-1} \).

A: (i) B: (i), (iii) C: (iii) D: (i), (ii) E: (ii)

A25. [2 marks] Exactly which of the following are cyclic groups?

(i) \( S_4 \), under binary operation composition of functions.
(ii) \( U_8 \), the group of units of \( \mathbb{Z}_8 \) under multiplication.
(iii) \( \mathbb{Q}^+ \), the positive rational numbers under multiplication.

A: (i), (ii) B: (i), (iii) C: (ii), (iii) D: (ii) E: None of A, B, C, D

B1. [4 marks] Prove that if \( G \) is a finite group containing an element of order 2, then \( |G| \) is even.

B2. [4 marks] Use mathematical induction to prove that for any integer \( n \geq 2 \),
\[
\sum_{k=2}^{n} \binom{k}{2} = \binom{n+1}{3}.
\]

B3. Let \( A, B, \) and \( C \) be sets and \( f:A \rightarrow B, g:B \rightarrow C \) and \( h:C \rightarrow A \) be functions such that \( h \circ g \circ f = 1_A \).

(a) [3 marks] Prove that \( f \) is injective.
(b) [3 marks] Prove that \( h \) is surjective.

B4. [3 marks] Give an example (with proof) of a relation \( R \) on \( J_3 \) which is not reflexive, not symmetric, not antisymmetric, but is transitive.
B5. [4 marks] Prove that for any sets $A$, $B$ and $C$, if $A \cap B = \emptyset$ and $B \cap C = \emptyset$, then $A \cap (B \cup C) \subseteq A \cap B \cap C$.

B6. [4 marks] Let $*$ denote the binary operation defined on $J_4$ by the table shown to the right. Prove that $*$ is not associative.

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B7. [4 marks] Find all left cosets of the subgroup $H = \{ ( \begin{array}{ccc} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{array} ) , ( \begin{array}{ccc} 2 & 1 & 3 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{array} ) \}$ of $S_3$ (with binary operation being composition of functions). Specifically, list every element of each left coset. 

B8. [5 marks] $U_{15}$, the group of units of $\mathbb{Z}_{15}$, has multiplication table:

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Find all subgroups of $U_{15}$. Each subgroup should be presented as a list of elements.

B9. [3 marks] Determine whether or not the function $f: (\mathbb{Z}, +) \rightarrow (\mathbb{Z}, +)$ defined by $f(n) = n^2$ is a homomorphism. Provide reasons for your answer.

B10. Let $R$ be the partial order relation on $J_3$ with Hasse diagram

Let $F$ denote the set of all functions from $J_2$ to $J_3$, and let $S$ denote the relation on $F$ given by

$S = \{ (f, g) \in F \times F \mid f(i) \leq_R g(i), \text{ for each } i = 1, 2 \}$.

(a) [3 marks] Prove that $S$ is a partial order relation on $F$.

(b) [3 marks] Draw a Hasse diagram for the partial order relation $S$ on the set $F$. For each $f = (\begin{array}{c} 1 \\ a \\ b \end{array}) \in F$, use the label $ab$ to represent $f$ in the diagram (for example, the function $(\begin{array}{c} 1 \\ 2 \\ 3 \end{array})$ would be represented by the label $11$).

B11. [3 marks] Draw the Hasse (poset) diagram for $D_{54} = \{ 1, 2, 3, 6, 9, 18, 27, 54 \}$, the set of positive divisors of 54, partially ordered by divisibility (that is, with partial order relation $R$ defined by $(a, b) \in R$ if $b = ka$ for some integer $k$). Label each vertex.

B12. [4 marks] Suppose that $f: (\mathbb{Z}_{50}, +) \rightarrow (\mathbb{Z}_{50}, +)$ is a homomorphism with $f(7) = 13$. Determine $f(1)$.

Bonus: [3 marks] In the symmetric group $S_5$, how many elements have order 5?
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Mathematics 2156A/B - Discrete Structures II

This course continues the development of logical reasoning and proofs begun in Mathematics 2155A/B. Topics include elementary number theory (gcd, lcm, Euclidean algorithm, congruences, Chinese remainder theorem) and graph theory (connectedness, complete, regular and bipartite graphs; trees and spanning trees, Eulerian and Hamiltonian graphs, planar graphs; vertex, face and edge colouring; chromatic polynomials).

Antirequisite(s):
Prerequisite(s): Mathematics 2155A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 4 lecture hours, 0.5 course.
Mathematics 2156b Course Outline (2010): Discrete Structures II

Instructor:

S. A. Rankin.

Textbook:


Prerequisites:

- Mathematics 2155, or the former Mathematics 222.

Course Outline:

- Number theory: greatest common divisors and Euclid's algorithm, prime numbers and the fundamental theorem of arithmetic, congruences, encryption.
- Graph theory: subgraphs, isomorphism, trees, Euler circuits, planar graphs, Hamilton cycles, graph colouring and chromatic polynomials.

Note: Topics may be added or deleted as time permits.

Evaluation of Student Performance:

- Graded homework, for a total of 10% of the final mark.
- Two midterm tests, each worth 20% of the final mark.
- Final examination, worth 50% of the final mark.

Examination Dates:

- First midterm, 9:30am-12:00noon, Saturday, January 30, SH 2355.
- Second midterm, 9:30-12:00noon, Saturday, March 6, 2010, SH 2355.
- Final exam: to be scheduled by the Registrar.
A1. [2 marks] The number of positive divisors of \((27)(49)(32)\) is:

\[\begin{array}{c|c|c|c|c|c|c}
A & 24 & B & 36 & C & 72 & D & 90 & E & \text{None of A, B, C, D} \\
\end{array}\]

A2. [2 marks] When the Euclidean algorithm is applied to a pair of (unknown) positive integers \(a\) and \(b\), the results are as shown at right. Based on this information the greatest common divisor of \(a\) and \(b\) can be written as:

\[
a = 4b + r_1 \\
b = 3r_1 + r_2 \\
r_1 = 4r_2 + r_3 \\
r_2 = 6r_3
\]

\[\begin{array}{c|c|c|c|c|c|c}
A & 13a - 56b & B & -3a + 11b & C & 12a - 22b & D & 4a - 19b & E & 13a - 69b \\
\end{array}\]

A3. [2 marks] When applying the Euclidean algorithm to find the gcd of a certain (unknown) number \(a\) and \(b = 66\), it is found that \(a = 66q_1 + 21\). What is \((a, 66)\)?

\[\begin{array}{c|c|c|c|c|c|c}
A & 1 & B & 3 & C & 4 & D & 21 & E & \text{None of A, B, C, D} \\
\end{array}\]

A4. [2 marks] If \(a = 24\), \((a, b) = 6\), and \([a, b] = 72\), then \(b\) is:

\[\begin{array}{c|c|c|c|c|c|c}
A & 8 & B & 48 & C & 18 & D & 36 & E & \text{None of A, B, C, D} \\
\end{array}\]

A5. [2 marks] Exactly which of \([6]_{22}, [12]_{22}, [15]_{22}\) are units in \(\mathbb{Z}_{22}\)?

\[\begin{array}{c|c|c|c|c|c|c}
A & [6]_{22} & B & \text{None of them} & C & [12]_{22}, [15]_{22} & D & \text{All of them} & E & \text{None of A, B, C, D} \\
\end{array}\]

A6. [2 marks] The number of integer values \(x\) such that \(35x \equiv 15 \pmod{60}\) with \(0 \leq x < 60\) is:

\[\begin{array}{c|c|c|c|c|c|c}
A & 0 & B & 1 & C & 5 & D & 7 & E & \text{None of A, B, C, D} \\
\end{array}\]

A7. [2 marks] How many values of \(x\) with \(0 < x \leq 36\) are solutions to the following system of linear congruences:

\[
\begin{align*}
2x & \equiv 8 \pmod{12} \\
 x & \equiv 7 \pmod{9}
\end{align*}
\]

\[\begin{array}{c|c|c|c|c|c|c}
A & 0 & B & 1 & C & 2 & D & 5 & E & \text{None of A, B, C, D} \\
\end{array}\]

A8. [2 marks] Exactly which of these statements are true for all positive integers \(a, b, c, \) and \(d\)?

(i) \(a\) divides \(b\) and \(c\) divides \(d\) implies that \(ac\) divides \(bd\).

(ii) \(b\) divides \(c\) implies that \(a + b\) divides \(a + c\).

(iii) \(a\) divides \(bc\) implies that \(a + b\) divides \(b\) or \(a\) divides \(c\).

\[\begin{array}{c|c|c|c|c|c|c}
A & (i) & B & (ii) & C & (iii) & D & (i), (ii) & E & (ii), (iii) \\
\end{array}\]

A9. [2 marks] The last decimal digit of \(2^{408}\) is:

\[\begin{array}{c|c|c|c|c|c|c}
A & 8 & B & 6 & C & 4 & D & 2 & E & 0 \\
\end{array}\]

A10. [2 marks] The size of a complete collection of trees on 5 vertices is:

\[\begin{array}{c|c|c|c|c|c|c}
A & 7 & B & 6 & C & 5 & D & 3 & E & \text{None of A, B, C, D} \\
\end{array}\]

A11. [2 marks] How many spanning trees does \(K_4\) have?

\[\begin{array}{c|c|c|c|c|c|c}
A & 15 & B & 8 & C & 14 & D & 12 & E & 16 \\
\end{array}\]
A12. [2 marks] Exactly which of the graphs $G_1, G_2, G_3, G_4$ shown above are regular?

| A: $G_1, G_4$ | B: $G_2, G_3$ | C: $G_2, G_4$ | D: $G_1, G_3$ | E: None of A, B, C, D |

A13. [2 marks] Exactly which of the graphs $G_1, G_2, G_3, G_4$ shown above are bipartite?

| A: $G_1$ | B: $G_2$ | C: $G_3$ | D: $G_4$ | E: None of A, B, C, D |

A14. [2 marks] Exactly which of the graphs $G_1, G_2, G_3, G_4$ shown above are eulerian?

| A: $G_1$ | B: $G_2$ | C: $G_3$ | D: $G_4$ | E: None of A, B, C, D |

A15. [2 marks] Exactly which of the graphs $G_1, G_2, G_3, G_4$ shown above are semi-eulerian?

| A: $G_1$ | B: $G_2$ | C: $G_3$ | D: $G_4$ | E: None of A, B, C, D |

A16. [2 marks] Exactly which of the graphs $G_1, G_2, G_3, G_4$ shown above are hamiltonian?

| A: None of them | B: None of them | C: $G_2, G_3$ | D: $G_1, G_4$ | E: $G_3, G_4$ |

A17. [2 marks] Exactly which of the graphs $G_1, G_2, G_3, G_4$ shown above are not planar?

| A: $G_1$ | B: $G_2$ | C: $G_3$ | D: $G_4$ | E: None of A, B, C, D |

A18. [2 marks] Exactly which of the graphs $G_1, G_2, G_3, G_4$ shown above have chromatic number equal to 3?

| A: $G_1, G_2$ | B: $G_1, G_3$ | C: $G_1, G_4$ | D: $G_2, G_3$ | E: $G_2, G_4$ |

A19. [2 marks] Exactly which of the graphs $G_1, G_2, G_3, G_4$ shown above have chromatic index equal to 3?

| A: $G_1, G_2$ | B: $G_1, G_3$ | C: $G_1, G_4$ | D: $G_2, G_3$ | E: $G_2, G_4$ |

A20. [2 marks] Which of the following polynomials are chromatic polynomials?

(i) $k^4 - 5k^3 + 4k$
(ii) $k^4 - 2k^3 + 3k^2$
(iii) $k^4 - 7k^3 + 6k^2$
(iv) $k^4 - k^3$

| A: (iii) | B: (iv) | C: (i) | D: (ii) | E: None of A, B, C, D |

A21. [2 marks] Let $G$ be a connected simple 4-regular plane graph. If $G$ has 8 faces, then how many vertices does $G$ have?

| A: 18 | B: 12 | C: 10 | D: 6 | E: None of A, B, C, D |

A22. [2 marks] Let $G$ be a plane graph. If $H$ is a natural plane embedding of the geometric dual $G^*$ of $G$, then exactly which of the following statements are true?

(i) $H$ is connected.
(ii) If $G$ is connected, then $H$ is connected.
(iii) If $H$ is connected, then $G$ is connected.
(iv) $|F_G| = |V_H|$.
(v) $|V_G| = |F_H|$.
A23. [2 marks] If $G$ is a simple graph with chromatic polynomial $P_G(k) = k(k-1)(k^2-3k+3)^2$, then the number of edges in $G$ is:

| A: 15 | B: 14 | C: 9 | D: 7 | E: 6 |

A24. [2 marks] What is the chromatic number of the graph shown at the right?

| A: 3 | B: 2 | C: 4 | D: 1 | E: None of A, B, C, D |

A25. [2 marks] What is the weight of an optimal spanning tree for the weighted graph shown to the right?

| A: 49 | B: 50 | C: 48 | D: 47 | E: 51 |

B1. [5 marks] Solve the following system of simultaneous congruences:

\[
\begin{align*}
    x &\equiv 3 \pmod{9} \\
    x &\equiv 6 \pmod{10} \\
    x &\equiv 7 \pmod{11}.
\end{align*}
\]

B2. (a) [3 marks] Solve $77x \equiv 1 \pmod{100}$.

(b) [3 marks] Find the last two digits (base 10) of $77^{279}$. Use Euler’s theorem and establish your answer.

B3. [4 marks] Prove that if $a$ and $b$ are positive integers with $(a, b) = 1$, then $(a - b, a + b)$ is either 1 or 2.

B4. [4 marks] Draw a complete collection of Eulerian simple graphs on 5 vertices.

B5. [5 marks] Calculate $\tau(G)$, the number of spanning trees of the graph $G$ shown below. Show your work.

B6. Consider the graph $G$:

(a) [2 marks] Number the edges sequentially in the diagram shown below to display an Euler trail in $G$ or else prove that no such trail exists.
(b) [2 marks] Either shade in appropriate edges in the diagram shown below to display a Hamilton cycle in $G$ or prove that no such cycle exists.

(c) [2 marks] Use the diagram of $G$ shown with dashed edges below to draw a natural plane embedding of the dual $G^*$.

(d) [3 marks] What is the chromatic number of $G$? Establish your answer.

(e) [6 marks] Calculate the chromatic polynomial $P_G(k)$.

B7. [4 marks] Find the chromatic index of the graph $G$ shown at the right:

B8. (a) [3 marks] Prove that the Herschel graph $G$ (shown below) does not contain a Hamilton cycle.

(b) [1 mark] Show that the addition of a suitable edge creates a Hamilton cycle. Use solid lines on the sketch below to show the edges in your cycle, including your new edge.

B9. [3 marks] Draw three connected simple 3-regular plane graphs with 8 vertices, no two of which are isomorphic. Each edge should be drawn as a straight line segment.
Bonus: [3 marks] Prove that if $G$ is a simple graph in which one vertex has degree 2 while all others have degree 3, then $\chi'(G) = 4$. 

**INSTRUCTOR & COURSE EVALUATION**
THE UNIVERSITY OF WESTERN ONTARIO

**Instructor Name**: Rankin, Stuart
**Teaching Faculty**: Science
**Teaching Department**: Mathematics
**Subject**: Mathematics
**Course Number**: 21568
**Section**: 001
**Year**: 2008-09
**Term**: Fall/Winter
**Enrolment**: 63
**Responses**: 26

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1.4.18 Mathematics 2211

Mathematics 2211A/B - Linear Algebra

Linear transformations, matrix representation, rank, change of basis, eigenvalues and eigenvectors, inner product spaces, quadratic forms and conic sections. Emphasis on problem-solving rather than theoretical development. Cannot be taken for credit by students in honors Mathematics programs.

Antirequisite(s): Applied Mathematics 2811B, Mathematics 2120A/B, the former Mathematics 203b.

Prerequisite(s): Linear Algebra 1600A/B or Mathematics 1120A/B with a minimum mark of 70%, or the former Applied Mathematics 202a or 212a.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
1.4.19 Mathematics 2212

Mathematics 2212A/B - Complex Variables
Complex numbers, Cauchy-Riemann equations, elementary functions, integrals, Cauchy's theorem and integral formula and applications, Taylor and Laurent expansions, isolated singularities, residue theorem and applications. Cannot be taken for credit by students in honors Mathematics programs.

Antirequisite(s): Mathematics 3124A/B, Applied Mathematics 3811A/B, the former Mathematics 306a/b.

Prerequisite(s): Calculus 2302A/B or Calculus 2502A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
Instructor: Jeffrey Morton, MC121. Email: jmort9@uwo.ca

Office Hours: TBA

Lectures: Tues & Fri 1430h-1530h (P&AB-36) & Thursday 1330h-1430h (NCB 285)


Course Web page: http://www.math.uwo.ca/~jmorton9/courses/2212b

Topics:

Prerequisite(s): Calculus 250 or 280 parts A and B

Please note the following statement required by Senate regulations: Unless you have either the requisites for this course or written special permission from your Dean to enroll in it, you may be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisites.

Assignments: There will be an assignment about every three weeks.

Exams: There will be a midterm exam held in class, and a final exam to be scheduled. These will cover material similar to that in the assignments.

Grades: The final grade will be computed as follows:

- Assignments: 20%
- Midterm Exam: 30%
- Final Exam: 50%

Senate Regulations on Scholastic Offenses (Plagiarism and Cheating): Please note the following statement required by Senate regulations: Scholastic offenses are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offense, at the following Web site:


(more details on academic rules can be found in the Senate Academic Handbook: http://www.uwo.ca/univsec/handbook/)
Math 2212b - Final Exam - Apr 13, 2009

1. Use the definition of the derivative to find

\[ \frac{d}{dz}(iz + 3)^2 \]

2. For \( u(x, y) = e^x(x \cos(y) - y \sin(y)) \), use the Cauchy-Riemann equations to check that \( u \) is a harmonic function. Verify that \( u \) is the real part of the analytic function \( f(z) = ze^z \).

3. Show that

\[ \sin(z) = \frac{e^{iz} - e^{-iz}}{2i} \]

by finding Taylor series for both sides at \( z_0 = 0 \).

4. Find a branch of \( \ln(z^2 - 1) \) which is analytic inside the disk \( |z| < 1 \). Indicate where it is not analytic.

5. Find

\[ \int_{\gamma} \frac{z}{z^4 - 1} \, dz \]

where \( \gamma \) is the circle of radius 2 about 0 oriented clockwise.

6. If \( f(z) \) is an entire function, and \( f(z) = i \) whenever \( |z| = 1 \), what do you know about \( f(0) \)? What about \( f'(0) \)? How do you know this? (Hint: Generalized Cauchy Integral Formula)

7. Find the first three terms of the Taylor series for

\[ f(z) = e^{z^2}\cos(z) \]

8. Use the (generalized) Cauchy Integral Formula to find

\[ \int_{\gamma} \frac{e^{\sin(z)}}{z^3} \, dz \]

where \( \gamma \) is the circle about the origin of radius 1, traversed once clockwise.
9 Describe all the singularities of
\[
\frac{7i}{z^2 + iz + 12}
\]

10 Where does the series
\[
\sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^{k+1} (z + 3)^k
\]
converge? Why?

11 Find the Laurent series for
\[
\frac{7i}{z^2 + iz + 12}
\]
about \(z_0 = 3i\). Where does it converge?

12 Find the trigonometric integral
\[
\int_{-\pi}^{\pi} \frac{1}{2 + \sin(\theta)} d\theta
\]
using the Residue Theorem.
## Instructor & Course Evaluation

### The University of Western Ontario

### Instructor Name

<table>
<thead>
<tr>
<th>Instructor Name</th>
<th>Teaching Faculty</th>
<th>Teaching Department</th>
<th>Subject</th>
<th>Course Number</th>
<th>Section</th>
<th>Year</th>
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### Questionnaire Results

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### Course Status

- 90% or more: A
- 70% to 89%: B
- 50% to 69%: C
- 20% to 49%: D
- 19% or less: E

### Initial Level of Enthusiasm

- High: 5
- Medium: 4
- Low: 3

### Percent of Classes Attended

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### Total

- Total: 6
Mathematics 2251F/G - Conceptual Development of Mathematics

A survey of some important basic concepts of mathematics in a historical setting, and in relation to the broader history of ideas. Topics may include: the evolution of the number concept, the development of geometry, Zeno's paradoxes.

Antirequisite(s): Philosophy 2251F/G.

Prerequisite(s): 1.0 course of university level Mathematics.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.

INSTRUCTOR & COURSE EVALUATION
THE UNIVERSITY OF WESTERN ONTARIO

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Percent of Classes Attended | Expected Grade | Course Status | Initial Level of Enthusiasm
90% or more | A | Required | High | 3
70% to 89% | B | Optional | Medium | 5
50% to 69% | C | Total | Low | 2
20% to 49% | D | Total | Total | 10
1.4.21 Mathematics 2290

Mathematics 2290 - Algebra
An introduction to abstract algebra, with principal emphasis on the structure of groups, rings, integral domains and fields. Cannot be taken for credit by students in honors Mathematics programs.
Antirequisite(s): Mathematics 3120A/B.
Prerequisite(s): 0.5 course from: Linear Algebra 1600A/B, Mathematics 2120A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 1.0 course.

1.4.22 Mathematics 2291

Mathematics 2291 - Elementary Theory of Numbers
Euclidean algorithm, congruences, indices, continued fractions, Gaussian integers, partitions and Diophantine equations.
Antirequisite(s): Mathematics 3150A/B.
Prerequisite(s): 1.0 course from: Calculus 1000A/B, 1100A/B, 1301A/B or 1501A/B, Applied Mathematics 1413, Mathematics 1120A/B, 1225A/B, 1228A/B, 1229A/B, Linear Algebra 1600A/B, the former Mathematics 030, 031.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 1.0 course.

1.4.23 Mathematics 2292

Mathematics 2292 - Synthetic Geometry
Groups of transformations of the Euclidean plane, inversion, the projective plane.
Antirequisite(s): Mathematics 4153A/B, the former Mathematics 319a/b.
Prerequisite(s): 1.0 course from: Calculus 1000A/B, 1100A/B, 1301A/B or 1501A/B, Applied Mathematics 1413, Mathematics 1120A/B, 1225A/B, 1228A/B, 1229A/B, Linear Algebra 1600A/B, the former Mathematics 030, 031.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 1.0 course.
Mathematics 2293 - Elementary Operations Research with Applications

Linear programming, basic probability and statistical distributions, networks, decision analysis, utility, game theory, inventory analysis, queuing theory, simulation, Markovian decision model, forecasting. Cannot be taken for credit by students in honors Mathematics programs.

**Antirequisite(s):** Applied Mathematics 3817A/B, Statistical Sciences 4654A/B, the former Statistical Sciences 236, 4737A/B.

**Prerequisite(s):** 1.0 course from: Calculus 1000A/B, 1100A/B, 1301A/B or 1501A/B, Applied Mathematics 1413, Mathematics 1120A/B, 1225A/B, 1228A/B, 1229A/B, Linear Algebra 1600A/B, the former Mathematics 030, 031. If Mathematics 1228A/B or the former Mathematics 031 is not taken, one of the following is also required, either as a prerequisite or a fall term co-requisite: Economics 2122A/B, 2222A/B, Statistical Sciences 2035, 2141A/B.
THE UNIVERSITY OF WESTERN ONTARIO  
LONDON                           CANADA  
DEPARTMENT OF MATHEMATICS  

Mathematics 2293  
2008-2009  

INSTRUCTOR: Gord Sinnamon  


PREREQUISITES: 1.0 course from: Calculus 1000A/B, 1100A/B, 1301A/B or 1501A/B, Applied Mathematics 1413, Mathematics 1120A/B, 1225A/B, 1228A/B, 1229A/B, Linear Algebra 1600A/B, the former Mathematics 030, 031. If Mathematics 1228A/B or the former Mathematics 031 is not taken, one of the following is also required, either as a prerequisite or a fall term co-requisite: Economics 2122A/B, 2222A/B, Statistical Sciences 2035, 2141A/B.  

ANTIREQUISITES: Applied Mathematics 3817A/B, Statistical Sciences 4737A/B, the former Statistical Sciences 236.  

COURSE OUTLINE: Linear programming, basic probability and statistical distributions, networks, decision analysis, utility, game theory, inventory analysis, queuing theory, simulation, Markovian decision model, forecasting. Cannot be taken for credit by students in honors Mathematics programs.  

WHAT IS EXPECTED OF THE STUDENT: Students are expected to attend all lectures and complete all quizzes, assignments and examinations. No aids are permitted in quizzes and examinations.  

EVALUATION OF STUDENT PERFORMANCE:  

1. In-Class Tests: 40% (Dates to be announced.)  
2. Written Assignments: 20%  
3. Final examination: 40%  

EXAMINATION DATES:  
The three-hour final examination will be given during the Final Examination period and will be scheduled by the Registrar’s Office.  

The Fine Print: If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean’s office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation for Special Examination" form must be obtained from the Dean's Office immediately. For further information please see: http://www.uwo.ca/univsec/handbook/appeals/medical.pdf. A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record’s Release Form (located in the Dean's Office) for visits to Student Health Services. The form can be found here: https://studentservices.uwo.ca/secure/medical_document.pdf. Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded and that all course prerequisites have been successfully completed. If the student does not have the requisites for a course, and does not have written special permission from his or her
Dean to enroll in the course, the student may be removed from the course and it will be deleted from the student’s record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites. Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site: http://www.uwo.ca/univsec/handbook/appeals/scholoff.pdf

Questions or comments about this page?
1. [10 marks] Solve the following linear programming problem using the simplex method. Show all your simplex tableaux. Write clear conclusions.

Maximize: \( z = x_1 + 4x_2 + 7x_3 \)
subject to: \( 2x_1 + 3x_2 + 5x_3 \leq 10 \)
\( 4x_1 - x_2 + x_3 \leq 10 \)
\( x_1 \geq 0, x_2 \geq 0, x_3 \geq 0. \)

2. [10 marks] ABC birdhouses is owned and operated by Ann, who does the sawing, Bill, who does the sanding and glueing, and Cathy, who does the painting. ABC is doing very well and has no problem selling its three products, the Wren House, the Pigeon Barn and the Magpie Manor. The time, in minutes, that each worker spends on one unit of each product is shown below along with the profit, in dollars, that ABC makes from one unit of each.

<table>
<thead>
<tr>
<th></th>
<th>Ann</th>
<th>Bill</th>
<th>Cathy</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wren House</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Pigeon Barn</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Magpie Manor</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Formulate a linear programming problem to find the shortest work day that ABC can have and how many of each product ABC should complete each day so that ABC will make at least $600 profit per day and so that each of Ann, Bill and Cathy will have enough time to finish their work. DO NOT SOLVE THE PROBLEM. Hint: Let one of the decision variables be the length of the work day in minutes.

3. Consider the following maximum problem.

Maximize: \( z = 7x_1 + 8x_2 + 10x_3 \)
subject to: \( 3x_1 + 6x_2 - 2x_3 = 14 \)
\( 2x_1 - x_2 + x_3 \geq 7 \)
\( x_1 \geq 0, x_2 \leq 0, x_3 \leq 0. \)

(a) [5 marks] Formulate the dual minimum problem.

(b) [5 marks] The optimal solution to the maximum problem above occurs at \( x_1 = 16/3, \) \( x_2 = -1/3, \) and \( x_3 = 0. \) Find the optimal value and the point \((y_1, y_2, y_3)\) at which the dual minimum problem achieves the optimal value.
\[ x_1 + x_2 + x_3 \leq 7. \] What is the new optimal value?

4. [15 marks] The Superior Oil Company has three oil refineries. Each refinery produces several products. The profit earned on the various products from the various refineries (in dollars per barrel) is shown below. Notice that some of the refineries are not equipped to produce some of the products, as indicated by the symbol N/A.

<table>
<thead>
<tr>
<th>Products</th>
<th>Refinery 1</th>
<th>Refinery 2</th>
<th>Refinery 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoline</td>
<td>10</td>
<td>8</td>
<td>N/A</td>
</tr>
<tr>
<td>Kerosene</td>
<td>N/A</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Diesel Fuel</td>
<td>7</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Jet Fuel</td>
<td>4</td>
<td>N/A</td>
<td>2</td>
</tr>
</tbody>
</table>

None of the refineries are currently working up to capacity. Refinery 1 has a capacity of 84,000 barrels but is currently producing only 75,000 barrels, Refinery 2 has a capacity of 62,000 barrels but is only producing 54,000 barrels, and Refinery 3 has a capacity of 120,000 barrels but is only producing 117,000 barrels.

Superior’s major customer would like Superior to supply some or all of the following in addition to what it already supplies: 6,000 barrels of gasoline, 7,000 barrels of kerosene, 5,000 barrels of diesel fuel, and 3,000 barrels of jet fuel.

Formulate and solve a balanced transportation problem to determine how much of the excess capacity at each refinery should be devoted to the production of each product to maximize Superior’s profit. State your conclusions clearly.

5. [10 marks] A project requiring activities A, B, C, D, E, F, G, and H has duration times and predecessor relations described in the table below:

<table>
<thead>
<tr>
<th>Activities</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Immediate Predecessors</td>
<td>–</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duration (weeks)</td>
<td>4</td>
<td>7</td>
<td>8</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Construct a CPM network diagram for this project, calculate early and late times for each node, and calculate the total float of each activity in the diagram. Clearly indicate all critical activities on the diagram. What is the completion time for the project?

6. Employees wishing to leave an environmentally controlled area join a single line beside the area’s four airlocks and wait for one to be free. Once an airlock is free it takes 20 seconds on average to get through. Actual times are exponentially distributed. On average, 6 employees arrive at the airlock line per minute, with interarrival times also exponentially distributed.

(a) [5 marks] For what proportion of the time are all the airlocks free?
(b) [5 marks] For what proportion of the time are all the airlocks busy?
on them at a time. Tourists arrive at random at the top of the stairs, where they wait in line for an average of 9 minutes before beginning their descent. It takes 3 minutes, on average, to descend the stairs with actual times being exponentially distributed.

(a) [5 marks] How many tourists use the stairs in an average hour?

(b) [5 marks] The stairs are to be replaced by a small elevator that takes exactly 3 minutes and 20 seconds to take one person down and return to the top. What will be the average wait at the top when this elevator is in use?

8. I must decide whether or not to buy a certain piece of land in the Sahara Desert. If I do buy the land and the proposed irrigation project is implemented I will make a profit of $200,000 but if I buy the land and the irrigation project is not implemented then I will lose $50,000. Sources inside the Egyptian Government tell me that there is a 75% chance of the project being implemented. For $4000 I can buy copies of records of recent purchases by the Egyptian Government. If the proposed project is going to be implemented then certain large purchases will appear in the records. However, even if the project is not going to be implemented, there is a 60% chance that those same purchases will appear anyway.

(a) [15 marks] Make a large, completely labeled decision tree to model this problem. What would you advise me to do?

(b) [5 marks] What is the most I should be willing to pay for copies of the purchase records?
## INSTRUCTOR & COURSE EVALUATION

THE UNIVERSITY OF WESTERN ONTARIO

<table>
<thead>
<tr>
<th>Instructor Name</th>
<th>Teaching Faculty</th>
<th>Teaching Department</th>
<th>Subject</th>
<th>Course Number</th>
<th>Section</th>
<th>Year Term</th>
<th>Enrolment</th>
<th>Responses</th>
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<td>Mathematics</td>
<td>Mathematics</td>
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<th>5</th>
<th>6</th>
<th>7</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Median</th>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>2. Well Organized</td>
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<td>5.75</td>
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<td>4</td>
<td>1</td>
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<td>1</td>
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<td>1</td>
<td>6</td>
<td>6.50</td>
<td>1.07</td>
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<td>9. Communicates Well</td>
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<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>5.88</td>
<td>0.83</td>
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<td>10. Adheres to Course Objectives</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<td>6.13</td>
<td>0.99</td>
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<td>0</td>
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<td>1</td>
<td>3</td>
<td>5.75</td>
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<td>0</td>
<td>3</td>
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<td>14. Good Motivator</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>5.25</td>
<td>1.16</td>
<td>5</td>
</tr>
<tr>
<td>Average of 1 to 14</td>
<td></td>
<td>5.88</td>
<td>1.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15. Overall Effectiveness</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>6.13</td>
<td>0.83</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>16. Course As Learning Experience</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>5.38</td>
<td>1.06</td>
<td>5.5</td>
</tr>
</tbody>
</table>
Mathematics 3020A/B - Introduction to Abstract Algebra

Sets and binary operations, commutativity, associativity, distributivity, groups and subgroups, cyclic groups, permutation groups, cosets, Lagrange’s theorem, normal subgroups, quotient groups, first isomorphism theorem, rings, integral domains, fields, polynomial rings, unique factorization of polynomials over a field, finite fields.

Antirequisite(s):  
Prerequisite(s): Mathematics 2120A/B or Applied Mathematics 2811B.

Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
Instructor.
Matthias Franz
Office: MC 134
E-mail: mfrazn at uwo dot ca

Class.
Tue 9:30–10:30 a.m. and Thu 9:30–11:30 a.m., MC 107
Website: http://www.math.uwo.ca/~mfrazn/courses/2009_3020a/

Textbook (recommended).
(free online access from within UWO)

Prerequisites.
Mathematics 2120A/B or Applied Mathematics 2811B

Course Description.
Math 3020 is intended as a preparation to more advanced pure math courses
in 3rd and 4th year. Learning to understand and to write mathematical proofs
is therefore an integral part of this course. At the same time, we\textsuperscript{1} will work out
many examples.

Assignments are an important tool both for learning how to write proofs and
for getting acquainted with examples. They encourage you to think about the
material, and the corrections give you the feedback you need to develop new
skills. There will be approximately one assignment per week. Homework will be
posted regularly on my webpage (see above) and collected in class on Tues-
days.

Syllabus.
Here is a brief outline of the topics I plan to cover.

\begin{itemize}
  \item The integers (axioms, induction, unique factorisation),
  \item Integers mod n (equivalence relations, Chinese remainder)
  \item Rings (definition, examples, polynomials, ideals and quotient rings, Chi-
    nese remainder)
  \item Groups (definition, examples, Lagrange's theorem, normal subgroups
    and quotients)
  \item Fields (finite field extensions, examples, extensions of the complex num-
    bers)
  \item If time permits: real division algebras and/or cardinal numbers
\end{itemize}

\textsuperscript{1}This is of course the politicians' "we", meaning "you".
Time: 180min; no aids allowed. Use the “blue” booklets as scrap paper and the “black” booklets for your answers. All answers must be justified unless stated otherwise. There are 50 marks in total.

1. (a) (2 marks) State the well-ordering principle.

(b) (3 marks) Using the well-ordering principle, prove that any natural number is a product of irreducible natural numbers.

2. (5 marks) Let \( m, n \in \mathbb{N} \) be coprime. Recall that there is a canonical ring morphism
\[
f : \mathbb{Z}_{mn} \to \mathbb{Z}_m \times \mathbb{Z}_n, \quad [c]_{mn} \mapsto ([c]_m, [c]_n).
\]
Pick \( s, t \in \mathbb{Z} \) such that \( sm + tn = 1 \). Show that the map
\[
g : \mathbb{Z}_m \times \mathbb{Z}_n \to \mathbb{Z}_{mn}, \quad ([a]_m, [b]_n) \mapsto [t(a) + s(b)]_{mn}
\]
is a well-defined morphism of rings which is inverse to \( f \).

3. How many solutions \( x \in \mathbb{Z}_{77} \) do the following equations have?
   (a) (2 marks) \( 11x = 7 \),
   (b) (4 marks) \( x^{120} = 1 \).
   Hint: How many units does \( \mathbb{Z}_{77} \) have?

4. (2 marks) Let \( R \) be an integral domain. Prove that any prime in \( R \) is irreducible.

5. (a) (2 marks) State the definition of a UFD.

(b) (2 marks) Give an example of an integral domain \( R \) which is not a UFD, including two essentially different factorizations of some \( a \in R \).

Continued on back
(6) (4 marks) Let $F$ be a finite field and $\Phi : F[X] \rightarrow \mathcal{F}(F, F)$ the map that sends a polynomial to the corresponding polynomial function. Show that the kernel of $\Phi$ is the principal ideal generated by
\[ f(X) = \prod_{a \in F} (X - a). \]

(7) (3 marks) Let $G$ be a commutative group. Show that the inversion map $\phi : G \rightarrow G, g \mapsto g^{-1}$ is an isomorphism of groups.

(8) Let $G = \{\pm 1, \pm i, \pm j, \pm k\} \subset \mathbb{H}$.
   (a) (3 marks) Show that $G$ is a group under multiplication.
   (b) (3 marks) Let $H = \{\pm 1, \pm j\} \subset G$. Verify that $H$ is a normal subgroup of $G$, and write down its cosets.
   (c) (2 marks) Is the quotient $G/H$ cyclic?

(9) (4 marks) Let $F$ be a field and $f(X) \in F[X]$ irreducible. Prove Kronecker’s theorem that there is a field extension of $F$ in which $f(X)$ has a root.

(10) Let $f(X) = X^3 + X - 1 \in \mathbb{F}_5[X]$.
    (a) (2 marks) Show that $f(X)$ is irreducible over $\mathbb{F}_5$.
    (b) (4 marks) Let $E$ be a field extension of $\mathbb{F}_5$ in which $f(X)$ has a root $\alpha$. Express $\alpha^{-1}$ and $(\alpha + 2)^{-1}$ as $\mathbb{F}_5$-linear combinations of 1, $\alpha$ and $\alpha^2$.

(11) (3 marks) List three distinct subfields $F$ of $\mathbb{C}$, $F \neq \mathbb{C}$. (No proof needed.)
## Instructor & Course Evaluation

### The University of Western Ontario

<table>
<thead>
<tr>
<th>Instructor Name</th>
<th>Teaching Faculty</th>
<th>Teaching Department</th>
<th>Subject</th>
<th>Course Number</th>
<th>Section</th>
<th>Year</th>
<th>Term</th>
<th>Enrolment</th>
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<td>Mathematics</td>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Median</th>
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<tbody>
<tr>
<td>1. Displays Enthusiasm</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
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<td>2</td>
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**Percent of Classes Attended**

- 90% or more: 7
- 70% to 89%: 2
- 50% to 69%: 0
- 20% to 49%: 0

**Expected Grade**

- A: 5
- B: 2
- C: 3
- D: 0

**Course Status**

- Required: 7
- Optional: 3
- Total: 10

**Initial Level of Enthusiasm**

- High: 5
- Medium: 3
- Low: 2
- Total: 10

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Mathematics 3120A/B - Group Theory

An introduction to the theory of groups: cyclic, dihedral, symmetric, alternating; subgroups, quotient groups, homomorphisms, cosets, Lagrange's theorem, isomorphism theorems; group actions, class equation, p-groups, Sylow theorems; direct and semidirect products, wreath products, finite abelian groups; Jordan-Hölder theorem, commutator subgroup, solvable and nilpotent groups; free groups, generators and relations.

Antirequisite(s):

Prerequisite(s): Mathematics 3020A/B.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 3120B

2009-2010

INSTRUCTOR: Lex Renner


PREREQUISITES: Mathematics 3020A.

COURSE OUTLINE: An introduction to the theory of groups. Groups, subgroups, permutations, cosets and direct products. Fundamental theorem of finitely generated abelian groups. Homomorphisms and factor groups. Sylow Theorems. Groups presentations. This material can be found in Chapters 1, 2, 3, and 7 of Fraleigh.

EVALUATION OF STUDENT PERFORMANCE:

Regular assignments worth 40% of the final mark.

Two in-class Midterm Examinations worth 10% each.

A three hour cumulative final examination, worth 40% of the final mark.

EXAMINATION DATES:

Assignment dates to be announced.

The Final Examination will be scheduled by the Registrar.

IMPORTANT SENATE POLICY:

Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded and that all course prerequisites have been successfully completed. If the student does not have the requisites for a course, and does not have written special permission from his or her Dean to enroll in the course, the student may be removed from the course and it will be deleted from the student's record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites.

STATEMENT ON ACADEMIC OFFENCES:

"Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically,
the definition of what constitutes a Scholastic Offence, at the following Web site:

NEW MEDICAL EXCUSE REGULATIONS:

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's Office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately.

For further information please see:


A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services.

The form can be found here:

https://studentservices.uwo.ca/secure/medical_document.pdf
1. (a) [4] State the Fundamental Theorem of Finitely Generated Abelian Groups.
   (b) [4] List all abelian groups (up to isomorphism) of order 216.

2. Let $G$ be a finite group acting on a finite set $X$.
   (a) [4] State Burnside’s formula.
   (b) [4] Illustrate the formula in (a) with $G = S_3$, $X = S_3$ and the action
       $$G \times X \to X, \ (g, x) \mapsto gxg^{-1}.$$
       i.e. Identify the three orbits and show how Burnside’s formula arrives at this number.

3. [8] State the three Sylow theorems.

4. Let $G$ be a group.
   (a) [4] Define what it means for $G$ to be simple.
   (b) [4] Suppose that $G$ is a finite group with $|G| = pq$, where $0 < p < q$ are primes. Prove that $G$ is not simple.
   (c) [4] Explain why a group of order 48 is not a simple group.

5. Let $G$ be a group.
   (a) [4] For $x, y \in G$ define $xRy$ if $gxg^{-1} = y$ for some $g \in G$. Prove that $R$ is an equivalence relation on $G$.
   (b) [4] Find a representative for each equivalence class of $S_4$.

6. Consider the group $G = S_n$. 1
(a) [4] Explain why any permutation $\sigma \in G$ is a product of transpositions.

(b) [4] Express $(1, 2, 3, 4)(5, 6) \in S_6$ as a product of transpositions.

7. Let $\varphi : G \to H$ be a homomorphism of groups.

(a) [4] Define $\ker(\varphi)$.

(b) [4] Suppose that there exists $x, y \in G$ and $\varphi(x) = \varphi(y)$. Prove that there exists $g \in \ker(\varphi)$ such that $y = gx$.

8. [4] What is the order of the group $(\mathbb{Z}_{12} \times \mathbb{Z}_{18})/ \langle (3, 3) \rangle$? Give a reason for your answer.

Answer any two of the remaining four problems.


10. [8] Let $G$ be a finite $p$-group acting on the finite set $X$. Prove that $|X_G| \equiv |X| \mod(p)$.

11. [8] Let $G$ be a group and let $H, K$ be normal subgroups of $G$ such that $H \cap K = \{e\}$. Prove that $hk = kh$ for all $h \in H$ and $k \in K$.

12. [8] Let $G = GL_n(\mathbb{R})$. Let $H = \{g \in G \mid \det(g) > 0\}$. Prove that $H$ is a normal subgroup of $G$. 
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1.4.27 Mathematics 3122

Mathematics 3122A/B - Metric Space Topology

An introduction to the theory of metric spaces with emphasis on the point-set topology of Euclidean n-space, including convergence, compactness, completeness, continuity, uniform continuity, homeomorphism, equivalence of metrics, connectedness, path-connectedness, fixed-point theorem for contractions, separability, complete normality, product spaces, category.

Antirequisite(s):

Prerequisite(s): Either Mathematics 2122A/B or Mathematics 2123A/B, each with a minimum mark of 60%.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
Lectures: Mon, Fri 10:30-11:45am, MC 107

Instructor: Janusz Adams
Office: MC 116
E-mail: jadams@uwo.ca
Office Hours: Thu, 1:00-3:00pm
Course Website: http://www.math.uwo.ca/~jadams/09-10/3122F2009/


Prerequisites: Minimum 60% in one of Mathematics 2122A/B or 2123A/B.

Course outline: An introduction to the theory of metric spaces with emphasis on the point-set topology of Euclidean n-space including convergence, compactness, completeness, continuity, uniform continuity, homeomorphism, equivalence of metrics, connectedness, path-connectedness, fixed-point theorem for contractions, separability, complete normality, product spaces, category.

Evaluation: The course mark will be based on 10 weekly quizzes (10 min. each), 2 term tests (approx. 60 min. each) held in class, and the final exam, with the following weights:

- Best 8 out of 10 quizzes: 24%
- Term tests: 36%
- Final exam: 40%

Examination dates: The term tests will be held on Oct.16 and Nov.13, 2009. Final exam TBA.

Problem Sets: The problem sets will be assigned weekly. These will not be submitted for marking. However, they are essential to the understanding of the material and preparation for the quizzes and tests.

Quizzes: The quizzes will be based on the material of the recently assigned problem sets.

Medical Excuse Regulations: Please note the following points, which are required to be stated in this outline.

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean’s office as soon as possible and contact your instructor immediately. It is the student’s responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a “Recommendation of Special Examination” form must be obtained from the Dean’s Office immediately.

A student requiring academic accommodation due to illness should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record’s Release Form (located in the Dean’s Office) for visits to Student Health Services.
1. (a) State the definition of a metric space.
    (b) Give an example of three metrics on \( \mathbb{R}^2 \) such that no two of them are topologically equivalent. Justify your answer.

2. Let \( \tau \) denote the Zariski topology on \( \mathbb{R} \), and let \( d \) denote the Euclidean metric in \( \mathbb{R}^3 \). Prove that every continuous mapping \( f : (\mathbb{R}, \tau) \to (\mathbb{R}^3, d) \) is bounded.

3. Prove or disprove the following statements:
   (a) If \((X, d)\) is a compact metric space, then any two disjoint closed sets \( A \) and \( B \) in \( X \) have positive distance (i.e., \( \text{dist}(A, B) := \inf \{d(a, b) | a \in A, b \in B\} > 0 \)).
   (b) If \( A \) and \( B \) are non-empty subsets of a metric space \((X, d)\) such that \( A \cup B \) and \( A \cap B \) are connected, then \( A \) and \( B \) are connected.

4. Let \( f : (V, \|\cdot\|) \to (\mathbb{Q}, \|\cdot\|) \) be a continuous mapping from a normed vector space \( V \) to the rational numbers (with Euclidean metric). Prove that \( f \) is constant.

5. Let \( d \) be a metric on a set \( X \), such that the induced topology \( \tau_d \) on \( X \) contains more than two elements. Prove that if \((X, d)\) is a connected metric space, then \( \tau_d \) contains infinitely many elements.

6. [Bonus]
   (a) Prove that every bounded subset of \( \mathbb{R}^n \) (with standard metric) is totally bounded.
   (b) Give an example of a bounded metric space which is not totally bounded.
**INSTRUCTOR & COURSE EVALUATION**

**THE UNIVERSITY OF WESTERN ONTARIO**

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**Percent of Classes Attended**
- 90% or more: 4
- 70% to 89%: 1
- 50% to 69%: 0
- 20% to 49%: 0

**Expected Grade**
- A: 1
- B: 4
- C: 0
- D: 0

**Course Status**
- Required: 4
- Optional: 0
- Total: 4

**Initial Level of Enthusiasm**
- High: 3
- Medium: 2
- Low: 0
- Total: 5
Mathematics 3123A/B - Differential Equations


**Antirequisite(s):**

**Prerequisite(s):** 2.0 courses: Calculus 2503A/B; Mathematics 2123A/B or the former Mathematics 306a/b; Mathematics 3122A/B; Mathematics 2120A/B; or Applied Mathematics 2811B or the former Mathematics 203b.

**Corequisite(s):**

**Pre-or Corequisite(s):**

**Extra Information:** 3 lecture hours, 0.5 course.
Mathematics 3123B, Winter 2009: DIFFERENTIAL EQUATIONS

Lectures: MWF 11:30-12:30, in MC 107

Instructor: Tatyana Foth (tfoth@uwo.ca)


Prerequisites: 2.0 courses: Calculus 2503A/B; Mathematics 2123A/B or the former Mathematics 306a/b; Mathematics 3122A/B; Mathematics 2120A/B; or Applied Mathematics 2811B or the former Mathematics 203b.

Antirequisite(s): none.


What is expected of the student: Students are expected to attend all lectures and complete all tests, assignments and examinations.

Evaluation of student performance:
1. Tests and homework assignments 60 %
2. Final examination 40 % (Tuesday, April 21, 9:00 am - 12:00 noon, according to a preliminary exam schedule)
If tests or assignments are missed due to a medical excuse and involve no more than 10% of the course mark, then the weight of missed coursework will be moved to the 60% tests and assignments component of the mark (it will be rescaled accordingly). For course work missed due to a medical excuse and involving more than 10% of the course mark, see the standard policy below.

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean’s office as soon as possible and contact your instructor immediately. It is the student’s responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean’s Office immediately. For further information please see:


A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record’s Release Form (located in the Dean’s Office) for visits to Student Health Services. The form can be found here:

https://studentservices.uwo.ca/secure/medical_document.pdf
Final exam (40 marks) Tuesday April 21, 9:00 am - 12:00 noon

In Problems 1-6 write a complete solution, show all steps.

In questions 7-14 no partial credit will be given, the correct answer is worth 1 mark, you do not have to show your work or explain your answer.

Problem 1. (4 marks) Solve the initial value problem

\[ \frac{dy}{dt} - 2y = t^3 \cos t, \quad t > 0 \]

\[ y(\pi) = \pi^2 \]

Problem 2. (5 marks) For the system

\[ \frac{dx}{dt} = x + 5y \]

\[ \frac{dy}{dt} = 3x + 3y \]

classify the critical point (0, 0) as to type, determine whether it is stable, asymptotically stable, or unstable. Sketch a phase portrait.

Problem 3. (7 marks) For the system

\[ \frac{dx}{dt} = x^2 - y \]

\[ \frac{dy}{dt} = \ln(1 - x + x^2) - \ln 3 \]

find all critical points, find the corresponding linear system near each critical point, classify each critical point as to type and determine whether it is stable, asymptotically stable, or unstable.

Problem 4. (6 marks) Find the general solution of the equation

\[ y^{(4)} - y = e^t + t^2 \]

Problem 5. (6 marks) Find the general solution of the equation

\[ t^2 y'' - 6y = 5t^3 + 8t^2, \quad t > 0 \]
Problem 6. (4 marks) For a power series solution \( \sum_{n=0}^{\infty} a_n x^n \) of the equation
\[
y'' + xy' + y = 0
\]
find the recurrence relation for the coefficients \( a_n \) and write at least the first six terms of the series (in terms of \( a_0, a_1 \)).

FOR PROBLEMS ON THIS PAGE CLEARLY INDICATE YOUR ANSWER BY CIRCLING "TRUE" or "FALSE".

7. Equation 
\[
(sin x + 2y)dx + (y \cos x + 2x)dy = 0
\]
is exact.

TRUE \hspace{1cm} FALSE

8. The set of solutions of the equation
\[
y''' + t^2 y' - y \cos t = 1, \quad -\infty < t < \infty
\]
is a 3-dimensional real vector space.

TRUE \hspace{1cm} FALSE

9. The system 
\[
\frac{dx}{dt} = x^3 y^2 \\
\frac{dy}{dt} = x^3 + y^3 + y
\]
has a nonconstant periodic solution.

TRUE \hspace{1cm} FALSE

PROBLEMS ON THIS PAGE ARE MULTIPLE CHOICE QUESTIONS. CIRCLE ONLY ONE ANSWER.

10. The solution \( r = 5, \ \Theta = \Theta_0 + t \) (here \((r, \Theta)\) are polar coordinates and \(\Theta_0\) is a real constant) of the system
\[
\frac{dr}{dt} = -r(r - 5)^2 \\
\frac{d\Theta}{dt} = 1
\]
is
\(\text{(A) an equilibrium solution}\)
(B) an unstable periodic solution
(C) a semistable limit cycle
(D) an asymptotically stable limit cycle
(E) none of the above

11. If
\[ y'' - xy' + (x + 1)y = 0 \]
\[ y(0) = 1 \]
\[ y'(0) = 0 \]
then \( y''(0) + y'''(0) \) is equal to
\[
\text{(A) } -2 \\
\text{(B) } -1 \\
\text{(C) } 0 \\
\text{(D) } 1 \\
\text{(E) none of the above}
\]

12. The Wronskian of a fundamental set of solutions of the equation
\[ t^2 y^{(4)} + ty''' + y' - 3y = 0, \ t > 0 \]
is
\[
\text{(A) } ce^{-t^2/2}, \text{ where } c \text{ is a non-zero constant} \\
\text{(B) } ce^{-t}, \text{ where } c \text{ is a non-zero constant} \\
\text{(C) } \frac{2}{t}, \text{ where } c \text{ is a non-zero constant} \\
\text{(D) } ce^{1/t}, \text{ where } c \text{ is a non-zero constant} \\
\text{(E) none of the above}
\]

PROBLEMS ON THIS PAGE ARE MULTIPLE CHOICE QUESTIONS. CIRCLE ONLY ONE ANSWER.

13. The number of the critical points of the system
\[ \frac{dx}{dt} = \sqrt{(x - y)^2 + 3 - 2} \]
\[ \frac{dy}{dt} = e^{y^2-x} - e \]
is
\[
\text{(A) } 0 \\
\text{(B) } 1 \\
\text{(C) } 2 \\
\text{(D) } 3 \\
\text{(E) none of the above}
\]
(B) 2  
(C) 3  
(D) 4  
(E) none of the above  

14. For the system  
\[
\begin{align*}
\frac{dx}{dt} &= x - y \\
\frac{dy}{dt} &= 8x
\end{align*}
\]

(A) the critical point \((0,0)\) is a spiral point and a typical trajectory has shape and direction as in (I)  
(B) the critical point \((0,0)\) is a spiral point and a typical trajectory has shape and direction as in (II)  
(C) the critical point \((0,0)\) is a spiral point and a typical trajectory has shape and direction as in (III)  
(D) the critical point \((0,0)\) is a spiral point and a typical trajectory has shape and direction as in (IV)  
(E) the critical point \((0,0)\) is not a spiral point
## Instructor & Course Evaluation

### The University of Western Ontario

<table>
<thead>
<tr>
<th>Instructor Name</th>
<th>Teaching Faculty</th>
<th>Teaching Department</th>
<th>Subject</th>
<th>Course Number</th>
<th>Section</th>
<th>Year</th>
<th>Term</th>
<th>Enrolment</th>
<th>Responses</th>
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<td>Mathematics</td>
<td>Mathematics</td>
<td>3123B</td>
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### Questions and Responses

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### Course Information

- **Expected Grade:**
  - A: 4
  - B: 1
  - C: 0
  - D: 0

- **Course Status:**
  - Required: 2
  - Optional: 3
  - Total: 5

- **Initial Level of Enthusiasm:**
  - High: 2
  - Medium: 3
  - Low: 0
  - Total: 5

### Attendance

- **Percent of Classes Attended:**
  - 90% or more: 5
  - 70% to 89%: 0
  - 50% to 69%: 0
  - 20% to 49%: 0
Mathematics 3124A/B - Complex Analysis I

The Cauchy-Riemann equations, elementary functions, branches of the logarithm and argument, Cauchy's integral theorem and formula, winding number, Liouville's theorem and the fundamental theorem of algebra, the identity theorem, the maximum modulus theorem, Taylor and Laurent expansions, isolated singularities, the residue theorem and applications, the argument principle and applications.

**Antirequisite(s):** [Applied Mathematics 3811A/B](#).

**Prerequisite(s):** [Mathematics 2123A/B](#) or the former Mathematics 306a/b.

**Corequisite(s):**

**Pre-or Corequisite(s):**

**Extra Information:** 3 lecture hours, 0.5 course.
Lectures: Tuesday & Thursday, from 10:50 to 12:10, in room MC 108.
Instructor: Martin Pinsonnault
Office: MC 133
E-mail: mpinson@uwo.ca

Office Hours: TBA

Textbook: The course will be based largely, but not exclusively, on the following book:


Prerequisite: Mathematics 2123A/B or the former Mathematics 306a/b.

Antirequisite: Applied Mathematics 3811A/B.

Course Outline: Complex numbers, the complex plane, functions of one complex variable, power series, the Cauchy-Riemann equations & analytic functions, line integrals, Cauchy’s theorem, winding number, Liouville’s theorem and the fundamental theorem of algebra, the identity theorem, the maximum modulus theorem, isolated singularities, Laurent series, the residue theorem, evaluation of definite integrals, harmonic functions. Further topics as time permits.

Evaluation: The evaluation will be based on

- weakly quizzes of 10 minutes each, for a total of 20% ;
- 2 term tests of approximately 50 minutes, held in class, for a total of 40% ;
- a comprehensive final exam 40%.

Examination dates: The term test will be held in mid February and mid March.

Problem sets: Problem sets will be assigned weakly but WILL NOT be marked. However, note that the quizzes and term tests will be partly based on those problem sets.
Q1: (5 marks) State the Cauchy Residue Theorem.

Q2: (5 marks) State Rouché’s Theorem.

Q3: (5 marks) State one version of Hurwitz’s Theorem.

Q4: (10 marks) Prove the Fundamental Theorem of Algebra using the Maximum (or Minimum) Modulus Principle.

Q5: (5 marks) Find the Laurent expansion for $f(z) = (z^4 - z^2)^{-1}$ about $z = 0$.

Q6: (5 marks) Classify the singularities of $f(z) = \exp(1/z^2)z^{-1}$.

Q7: a) (5 marks) Determine the singularities and residues of $f(z) = ze^{3/z}$.
   
   b) (5 marks) Evaluate $\int_{|z|=2} ze^{3/z} \, dz$.

Q8: a) (5 marks) Determine the singularities and residues of $f(z) = \sin(1/z)$.
   
   b) (5 marks) Evaluate $\int_{|z|=1} \sin(1/z) \, dz$.

Q9: (10 marks) Show that $\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \pi$ without computing an antiderivative for $\frac{1}{1+x^2}$.

Q10: (5 marks) Let $f(z) = z^5 - 3iz^2 + 2z - 1 + i$. Evaluate $\int_{C} \frac{f'(z)}{f(z)} \, dz$ where $C$ encloses all the zeroes of $f(z)$.

Q11: (5 marks) Let $f(z) = \frac{(z^2 + 1)^2}{(z^2 + 2z + 2)^3}$. Evaluate $\frac{1}{2\pi i} \int_{|z|=4} \frac{f'(z)}{f(z)} \, dz$.

Q12: (5 marks) Find the number of zeros of $f(z) = z^5 + 5z - 2$ in the annulus $1 \leq |z| \leq 2$.

Q13: (5 marks) Evaluate $\int_{C} ze^{3z^2} \, dz$ where $C$ is the curve joining $(1,1)$ and $(2,3)$ given
   
   by $y = x^3 - 3x^2 + 4x - 1$.

Q14: (10 marks) Suppose that an entire function satisfies the inequality $0 < |f(z)| < 4/e^2$ whenever $|z| = 2$. Prove that the function $\frac{f(z)}{e^z}$ has exactly one fixed point in the open disc $|z| < 2$.

Q15: (5 marks) Find a function with a simple pole of residue $1/\pi$ at each integer $n \in \mathbb{N}$.

Q16: (10 marks) Let $f$ be an analytic and \textit{bounded} in the right half-plane. Suppose $f$ extends continuously to the imaginary axis and satisfies $|f(it)| \leq M, t \in \mathbb{R}$. Show that $|f(z)| \leq M$ in the right half-plane. \textit{Hint:} Consider $(z+1)^{-s} f(z)$ on a large semidisc and apply the Maximum principle.

Total: (110 marks out of 100).
Mathematics 3132B - General Topology

Topological spaces, operations on subsets (e.g. closure), neighbourhoods, bases, subspaces, quotient spaces, product spaces, connectedness, compactness, countability and separation axioms, function spaces.

Antirequisite(s): The former Mathematics 4121A.

Prerequisite(s): Mathematics 3122A/B.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
Mathematics 3150

Mathematics 3150A/B - Elementary Number Theory I
Divisibility, primes, congruences, theorems of Fermat and Wilson, Chinese remainder theorem, quadratic reciprocity, some functions of number theory, diophantine equations, simple continued fractions.

Antirequisite(s): Mathematics 2291.

Prerequisite(s): 1.0 course in Mathematics, Applied Mathematics, Calculus, or Differential Equations at the 2100 level or higher.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS
Mathematics 3150A

INSTRUCTOR: Lex Renner, 130 Middlesex College

TEXTBOOK: Notes will be distributed.

PREREQUISITES: 1.0 course in Mathematics, Applied Mathematics, Calculus, or Differential Equations at the 2100 level or higher.

ANTIREQUISITES: Mathematics 2291

COURSE OUTLINE: Elementary number theory I: Divisibility, primes, congruences, theorems of Fermat and Wilson, Chinese remainder theorem, quadratic reciprocity, some functions of number theory, diophantine equations, simple continued fractions.

EVALUATION OF STUDENT PERFORMANCE:

Regular assignments worth 40% of the final mark.

Two in-class Midterm Examinations worth 10% (each) of the final mark.

One three hour cumulative final examination, worth 40% of the final mark.

EXAMINATION DATES:

Assignment dates to be announced.

The Final Examination will be scheduled by the Registrar.

IMPORTANT SENATE POLICY:

Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded and that all course prerequisites have been successfully completed. If the student does not have the requisites for a course, and does not have written special permission from his or her Dean to enroll in the course, the student may be removed from the course and it will be deleted from the student’s record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites.

STATEMENT ON ACADEMIC OFFENCES:

"Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically,
the definition of what constitutes a Scholastic Offence, at the following Web site:
http://www.uwo.ca/univsec/handbook/appeals/scholoff.pdf"

NEW MEDICAL EXCUSE REGULATIONS:

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's Office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office immediately.

For further information please see:


A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record’s Release Form (located in the Dean's Office) for visits to Student Health Services.

The form can be found here:

https://studentservices.uwo.ca/secure/medical_document.pdf
1. [8] Find all integer solutions to the following system of equations.
   \[ x \equiv 1 \mod(4) \]
   \[ x \equiv 2 \mod(5) \]
   \[ x \equiv 3 \mod(7). \]

2. (a) [2] Find \( g = (99, 143) \).
   (b) [3] Find integers \( x \) and \( y \) so that \( g = 99x + 143y \).

3. Consider the Diophantine equation \( x^2 + y^2 = z^2 \).
   (a) [4] Define what it means to be a primitive, positive solution to this equation.
   (b) [4] Find four distinct primitive, positive solutions with \( x \) odd and \( y \) even.

4. (a) [3] Let \( f \) and \( g \) be multiplicative functions. Define the Dirichlet product \( f \ast g \).
   (b) [3] Define the Moebius function \( \mu \).
   (c) [5] Calculate \( \mu \ast \tau \) where \( \tau(n) \) = the number of positive divisors of \( n \).

5. [8] Find all odd primes \( p \) such that \( x^2 + 2x - 4 \equiv 0 \mod(p) \) has a solution.

6. (a) [4] Characterize the set of odd rational primes \( p \) that split in \( \mathbb{Z}[\sqrt{2}] \).
   (b) [4] Using your criterion from (a) decide whether the following rational primes split in \( \mathbb{Z}[\sqrt{2}] \). (i) 7, (ii) 11, (iii) 29, (iv) 41.

7. Let \( p \) be a rational prime.
2 MA THEMATICS 3150A FINAL EXAMINATION

(a) [4] How many primitive roots mod(p) are there?
(b) [4] If a has order h mod(p) what is the order of \( a^k \) mod(p)?
(c) [4] How many elements mod(191) have order 95?

8. [8] Find all odd primes \( p \) such that \( \left( \frac{2}{p} \right) = -1 \).

9. (a) [3] State the Division Algorithm for \( \mathbb{Z}[\sqrt{2}] \).
(b) [3] Find the g.c.d. in \( \mathbb{Z}[\sqrt{2}] \) of \( 2 - \sqrt{2} \) and \( 6 - 2\sqrt{2} \). You should not need to use the result from part (a).

10. [6] Prove any two of the following assertions. State clearly any results you use from class notes.
(a) If \( (x,p) = 1 \) and \( \left( \frac{x}{p} \right) = -1 \) then \( x^{\frac{p-1}{2}} \equiv (p-1)! \mod(p) \).
(b) \( \left( \frac{-1}{p} \right) = 1 \) if and only if \( p \equiv 1 \mod(4) \).
(c) Let \( p \) be a rational prime and suppose that \( a^2 - 2b^2 = p \), where \( a, b \in \mathbb{Z} \). Then \( \left( \frac{2}{p} \right) = 1 \).
(d) Suppose that \( x, y \in \mathbb{Z} \) are both odd. Then \( x^2 + y^2 \) is not a perfect square.
(e) Let \( \mu \) be the Moebius function, \( \tau(n) = \) the number of positive divisors of \( n \), and \( U(n) = 1 \), for all \( n \). Then \( \mu \ast U \ast \tau = \tau \).
### INSTRUCTOR & COURSE EVALUATION
THE UNIVERSITY OF WESTERN ONTARIO

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### 1.4.32 Mathematics 3151

**Mathematics 3151A/B - Elementary Number Theory II**

Arithmetic functions, perfect numbers, the Möbius inversion formula, introduction to Dirichlet series and the Riemann zeta function, some methods of combinational number theory, primitive roots and their relationship with quadratic reciprocity, the Gaussian integers, sums of squares and Minkowski’s theorem, square and triangular numbers, Pell’s equation, introduction to elliptic curves.

**Antirequisite(s):** Mathematics 2291.

**Prerequisite(s):** Mathematics 3150A/B or Mathematics 2156A/B.

**Corequisite(s):**

**Pre-or Corequisite(s):**
Mathematics 3152A/B - Combinatorial Mathematics

Enumeration, recursion and generating functions, linear programming, Latin squares, block designs, binary codes, groups of symmetries, orbits, and counting.

Antirequisite(s):

Prerequisite(s): 0.5 course from Mathematics 2120A/B, 2156A/B, 2211A/B, Applied Mathematics 2811B or the former Mathematics 203b, or permission of the Department.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
INSTRUCTOR: Ján Mináč
OFFICE: 131 Middlesex College
OFFICE HOURS: To be announced.
TEXTBOOK: Notes from the class are sufficient. Additional literature will be recommended.
PREREQUISITES: 0.5 course from: Mathematics 2120A/B (old number 200a/b), 2156B (old number 223b), 2211B (old number 283b), Applied Mathematics 2811B (old number 213b) or the former Mathematics 203b, or permission of the Department.
COURSE OUTLINE: Math 3152A/9043A will be a course in enumerative combinatorics. Specific topics will include:

- counting arguments and the pigeonhole principle;
- permutations and combinations of sets and multisets;
- binomial and multinomial coefficients and Stirling numbers;
- variations on the inclusion-exclusion principle;
- ordinary and exponential generating functions;
- counting in the presence of symmetry: an introduction to group actions and Polya theory

EVALUATION OF STUDENT PERFORMANCE:
Evaluation will be discussed in class.

COMMENTS: Few things seem to be more boring than counting. One can imagine a person spending his days counting from 1 to one million and more. BRR!!! Now this course is about the Art of Counting. Ingenious, clever, surprising and fun activity. It is a magical door, a magnificent gate into the garden of modern mathematics. WELCOME TO THE PALACE OF COMBINATIONS!!
Mathematics 3152A/9043A Special Final Examination April 2009

In exercises 1, 2 and 3, \( G \) is a group with 21 elements. (It is convenient to observe that \( 21 = 7 + 7 + 7 = 3 \cdot 7 \).) These exercises taken together lead to a nice theorem, but they are independent from each other. This is the reason why I repeat as hypothesis some conclusions of previous exercises.

1. (a) [8 marks] Show that there is a unique 7-Sylow subgroup \( H \) of \( G \).
(b) Show that any 7-Sylow subgroup \( H \) of \( G \) is normal in \( G \).

2. (a) [8 marks] Show that there exists a subgroup \( B = \{1, b, b^2\}, \ b^3 = 1 \) of \( G \).
(b) Show that if \( H \) is a subgroup with 7 elements of \( G \) then \( H = \{1, a, \ldots, a^6\}, \ a^7 = 1 \), for some \( a \in G \).
(c) Let \( b \) be as in (a) and (b) and \( H \triangleleft G \) (\( H \) is normal in \( G \).) Using the fact that \( a = b^3a^{-3} \) show that we have one of the following:

(i) \( ba = ab \)
(ii) \( bab^{-1} = a^2 \)
(iii) \( bab^{-1} = a^4 \)

(d) Assume \( B \) and \( H \) are as in (a) and (b). Show that \( G = \{a^ib^j \mid 0 \leq i \leq 6, \ 0 \leq j \leq 2\} \).

3. [8 marks] Consider the following two matrices with entries in \( \mathbb{F}_7 \):

\[
A = \begin{pmatrix}
1 & 1 \\
0 & 1
\end{pmatrix}, \quad B = \begin{pmatrix}
4 & 0 \\
0 & 2
\end{pmatrix}
\]

Show that
(a) The group generated by \( A \) has order 7.
(b) The group generated by \( B \) has order 3.
(c) Show that the group \( G \) generated by \( A \) and \( B \) has order 21.
(d) Find all 7-Sylow subgroups of \( G \).

4. [16 marks] Use the Inclusion-Exclusion Principle to find the number of
\[
\{n \mid n \leq 210, 7 \nmid n \text{ and } 3 \nmid n \text{ and } 5 \nmid 210\}.
\]

5. [5 marks] Let \( \Phi_{12}(x) = \prod_{\text{order of } \zeta = 12} (x - \zeta) \).
(Order of \( \zeta = 12 \) means that \( \zeta^{12} = 1 \) but \( \zeta^i \neq 1 \) for all \( 1 \leq i \leq 11 \).)
Determine \( \Phi_{12}(x) \). (Please justify your solution carefully.)

6. [15 marks] Determine \( \varphi(\varphi(12)) \), where \( \varphi \) is the Euler function \( \varphi(n) = \#\{k \mid (k, n) = 1, 1 \leq k \leq n\} \).

7. [15 marks] Let \( \Pi \in S_{10} \) given as
\[
\Pi = (1, 2, 3)(4, 6)(7, 8, 9, 10)
\]
where \( (1,2,3) \), for example, is the cycle \( 1 \to 2 \to 3 \to 1 \) and \( (5) \) is the singleton, this means that it is a cycle \( 5 \to 5 \). Determine the order of \( \Pi \). (Carefully, as always, justify your solution.)

8. [5 marks] Determine for which \( n \in \mathbb{N} (1, 2, \ldots, n) \in S_n \) belong to \( A_n \). \( A_n \) is the alternating subgroup of \( S_n \). It has index 2 in \( S_n \).

9. (a) [10 marks] Determine the number of cubic irreducible polynomials over \( \mathbb{F}_5 \).
(b) Find one (explicit) irreducible cubic polynomial \( f(x) \) over \( \mathbb{F}_5 \). (Again please do not forget to carefully justify your solution.)

10. [10 marks] Let \( a_1 = 1 \) and \( a_2 = 3 \), and \( a_{n+2} = 2a_{n+1} - a_n, n \in \mathbb{N} \). Find a formula for \( a_n \) depending on \( n \). (Hint: Find a few first terms of this sequence.)
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**THE UNIVERSITY OF WESTERN ONTARIO**

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1.4.34 Mathematics 3153

Mathematics 3153A/B - Discrete Optimization

Antirequisite(s):
Prerequisite(s): One of: Mathematics 2156A/B, 3152A/B, the former Statistical Sciences 236, or permission of the Department.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

1.4.35 Mathematics 3154

Mathematics 3154A/B - Introduction to Algebraic Curves
Geometry of algebraic curves over the rational, real and complex fields. Classification of affine conics, singularities, intersection numbers, tangents, projective algebraic curves, multiplicity of points, flexes. Some discussion of cubic curves. 

Antirequisite(s): Mathematics 2292 and the former Mathematics 319a/b
Prerequisite(s): Linear Algebra 1600A/B or Mathematics 2120A/B; Mathematics 2121A/B, 2122A/B or 2155A/B; an additional 0.5 course in Mathematics, Applied Mathematics, Calculus, or Differential Equations at the 2100 level or above.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

1.4.36 Mathematics 3958

Mathematics 3958A/B - Special Topics in Mathematics

Antirequisite(s):
Prerequisite(s): Permission of the Department.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

1.4.37 Mathematics 3959

Mathematics 3959A/B - Special Topics in Mathematics

Antirequisite(s):
Prerequisite(s): Permission of the Department.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
Mathematics 4120

Mathematics 4120A/B - Field Theory

Automorphisms of fields, separable and normal extensions, splitting fields, fundamental theorem of Galois theory, primitive elements, Lagrange’s theorem. Finite fields and their Galois groups, cyclotomic extensions and polynomials, applications of Galois theory to geometric constructions and solution of algebraic equations.

Antirequisite(s):

Prerequisite(s): Mathematics 4123A/B, the former Mathematics 308a/b.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Mathematics 4120B/9020B

INSTRUCTOR: Ján Mináč


Lectures on Finite Fields and Galois Rings by Zhe-Xian Wan, published by World Scientific Publishing Co. Inc. (Optional)

PREREQUISITES: Enthusiasm, curiosity, and fun-loving attitude towards mathematics is very welcome.

COURSE OUTLINE:
Automorphisms of fields, separable and normal extensions; splitting fields, the fundamental theorem of Galois theory, primitive elements, Lagrange's theorem. Finite fields and their Galois groups, cyclotomic extensions and polynomials, applications of Galois theory to geometric constructions and the solution of algebraic equations.

COMMENT: Galois theory is one of the most beautiful, deep, and romantic subjects in science and art. It should be a great adventure and fun to investigate this subject.
EVALUATION OF STUDENT PERFORMANCE:
Will be discussed in class.

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https://studentservices.uwo.ca/secure/medical_document.pdf
Mathematics 4120B/9020B Final Examination Monday, April 13, 2009

1. (a) [15 marks] Show that $\mathbb{Q}(\sqrt{5})/\mathbb{Q}$ is a Galois extension.
(b) Find its Galois group $G$.
(c) Establish one-one correspondence between intermediate subfields $K$, $\mathbb{Q} \subset K \subset \mathbb{Q}(\sqrt{5})$ and subgroups of $G$.

2. (a) [5 marks] Show that $\mathbb{Q}(\sqrt{5}, \sqrt{11})$ is a Galois extension.
(b) Find its Galois group $G$.
(c) Establish one-one correspondence between intermediate subfields $K$, $\mathbb{Q} \subset K \subset \mathbb{Q}(\sqrt{5}, \sqrt{11})$ and subgroups of $G$.

3. (a) [10 marks] Find the splitting field $F$ of the polynomial $x^4 + x^2 - 1$ over $\mathbb{F}_3$.
(b) Determine the Galois groups $G = \text{Gal}(E/\mathbb{F}_3)$.
(c) Find the 1-1 correspondence between intermediate subfields $K$, $\mathbb{F}_3 \subset K \subset E$ and subgroups of $G$.

4. [5 marks] Let $p$ be a prime number, $f(x) \in \mathbb{F}_p[x]$ be an irreducible polynomial over $\mathbb{F}_p$ and $E$ be a splitting field of $f(x)$. Show that if $\alpha \in E$ is a root of $f(x)$ then for any other $\beta \in E$ there exists $k \in \mathbb{N}$ such that $\beta = \alpha^p^k$.

5. [15 marks] Determine all irreducible polynomials of degree 1, 2, 3 and 4 over $\mathbb{F}_2$. Please determine which cubic polynomials are separable.

6. [10 marks] Let $\zeta_p = \cos \frac{2\pi}{p} + i \sin \frac{2\pi}{p}$. Let $E = \mathbb{Q}(\zeta_p)$.
   (a) Determine $N_{E/\mathbb{Q}}(\zeta_p - 1)$ when $p = 3$.
   (b) Determine $N_{E/\mathbb{Q}}(\zeta_p - 1)$ when $p = 5$.

7. [10 marks] Let $\zeta_p = \cos \frac{2\pi}{p} + i \sin \frac{2\pi}{p}$, $p$ is a prime, $E = \mathbb{Q}(\zeta_p)$. Determine $\text{Tr}_{E/\mathbb{Q}}(\zeta_p)$.

8. [10 marks] Let $E/F$ be a Galois field extension such that $\text{Gal}(E/F) = S_3$. Determine whether there exist two distinct fields $K_1, K_2$ such that
   - $F \subset K_1, K_2 \subset E$
   - $K_1 \subset K_2$.

9. [10 marks] Let $E/F$ be a Galois extension with Galois group $G$. Assume that for each $\sigma \in G \setminus \{1\}$, $\sigma^2 = 1$ there is a $\tau \in G$ such that $\sigma \tau \neq \tau \sigma$. Does there exist an extension $K/F$ such that $K/F$ is Galois and $[E : K] = 2$?

10. [10 marks] Let $E$ be a splitting field of $x^4 - 17$ over $\mathbb{Q}$. Please determine $\text{Gal}(E/\mathbb{Q})$. 

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Mathematics 4122A/B - Lebesgue Integration and Fourier Series

Lebesgue measure, measurable sets and functions, approximation theorems, the Lebesgue integral, comparison with the Riemann integral, the basic convergence theorems and continuity properties, the space L2, the Riesz-Fischer theorem and completeness of the trigonometric system, pointwise convergence of Fourier series, Fejér's theorem.

Antirequisite(s):
Prerequisite(s): Mathematics 3122A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
INSTRUCTOR: A. Dhillon


PREREQUISITES: Mathematics 3122A/B (old number Mathematics 304a).

COURSE OUTLINE: Measure and integration theory on the real line: outer measure, measurable sets, Lebesgue measure and integral, convergence, completeness, and differentiation theorems, $L^p$-spaces, Haar measure.

WHAT IS EXPECTED OF THE STUDENT? To be announced in class.

EVALUATION OF STUDENT PERFORMANCE:
- Assignments 60%
- In-class Presentation 40%

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### Instructor & Course Evaluation

**The University of Western Ontario**

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Mathematics 4123A/B - Rings and Modules

Commutative rings, ring homomorphisms and quotient rings, ideals, rings of fractions, the Chinese remainder theorem; Euclidean domains, principal ideal domains, unique factorization domains; polynomial rings over fields; modules, direct sums of modules, free modules; modules over a principal ideal domain, the rational canonical form, the Jordan canonical form.

Antirequisite(s): The former Mathematics 308a/b.

Prerequisite(s): Mathematics 3120A/B.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
The following course will be offered in the Fall term of 2009-2010

Mathematics 4123A/9023A

Rings and Modules

| Instructor:      | Professor David Riley  
|                 | Department of Mathematics  
|                 | Middlesex College 118  
|                 | Telephone: (519) 661-3639  
|                 | Email: DMRiley@uwo.ca  
| Time and Location: | Tuesdays 9:30-10:30 and Thursdays 9:30-11:30 in MC 108 (starting on Tuesday, September 15)  

Prerequisites:
Math 3020A/B for Math 4123A. Special permission of the instructor is required for to enroll in Math 9023A. This course assumes a basic background in linear algebra and one introductory course in abstract algebra. Please note the following point, which is required to be stated in this outline by Senate regulation: "Unless you have either the prerequisites for this course or the written special permission of your Dean to enroll in it, you may be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event you are dropped from a course for failing to have the necessary prerequisites."

Synopsis:
The object of this course is to introduce the fundamental algebraic concept of a module over a ring. Theory will be developed in order to solve such problems as how to classify finitely generated Abelian groups and how to choose a basis to represent linear transformations as matrices with in easy and manageable canonical forms.

Main Topics (as time permits):
- Cartesian products and Zorn's Lemma
- A quick review of introductory ring theory
- Euclidean domains, principal ideal domains and unique factorization domains
- Polynomial rings
- Introduction to module theory
- Tensor algebras, symmetric and exterior algebras
- Modules over principal ideal domains
- Depending on time and interest, other topics may include basic category theory, Noetherian rings, Artinian rings, discrete valuation rings, and Dedekind domains

**Evaluation of Student Performance:**
- Five assignments, one roughly every two weeks, worth 10% of the final mark each
- One in-class midterm examination on October 22 worth 25% of the final mark
- One take-home final examination worth 25% of the final mark
- Performance expectations are significantly higher in Math 9023A than in Math 4123A

**Senate Regulations on Scholastic Offences (Plagiarism and Cheating):**
Please note the following points, which are required to be stated in this outline by Senate regulation: "Plagiarism is a major academic offence (see Scholastic Offence Policy in the Western Academic Calendar). The University of Western Ontario uses software for plagiarism checking. Students may be required to submit their written work in electronic form for plagiarism checking. In addition, if any computer-marked multiple-choice tests and/or exams are given, software to check for unusual coincidences in answer patterns that may indicate cheating may be used."
1.4.41 Mathematics 4151

Mathematics 4151A/B - Algebraic Number Theory

Algebraic numbers, cyclotomic fields, low dimensional Galois cohomology, Brauer groups, quadratic forms, local and global class fields, class field theory, Galois group representations, modular forms and elliptic curves, zeta function and L-series.

Antirequisite(s):

Prerequisite(s): Mathematics 4120A/B; Mathematics 3151A/B strongly recommended but not required.

Corequisite(s):

Extra Information: 3 lecture hours. 0.5 course.
Mathematics 4152A/B - Algebraic Topology

Homotopy, fundamental group, Van Kampen's theorem, fundamental theorem of algebra, Jordan curve theorem, singular homology, homotopy invariance, long exact sequence of a pair, excision, Mayer-Vietoris sequence, Brouwer fixed point theorem, Jordan-Brouwer separation theorem, invariance of domain, Euler characteristic, cell complexes, projective spaces, Poincaré theorem.

Antirequisite(s):
Prerequisite(s): Mathematics 3120A/B, the former Mathematics 4121A.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
MATHEMATICS 4152b/59052b
Course Outline -September 2009

Instructor: R. Kane  office: MC 136
  email: rkane@uwo.ca

Class: Monday 9:30-11:00am and Wednesday 9:30-11:00am in Middlesex
  College 108

Text: The text for the course is Algebraic Topology, by Allen Hatcher.
  Published by Cambridge University Press. ISBN 0-521-79540-0. The book
  is available online at

http://www.math.cornell.edu/~hatcher/AT/ATpage.html

It can be downloaded free of charge. I will order copies of the book through
the bookstore for those who are interested. The paperback version is
currently selling for $37US.

Prerequisites: Group Theory (Math 3120a) and General Topology (Math
  4121a)

Evaluation of Student Performance: Final grades will be determined as
follows:
  (i) Midterm   (40%)
  (ii) Assignments  (20%)
  (iii) Final exam   (40%)

Course Content: Here are the main topics that I hope to cover in the
course: Homotopy, fundamental group, Van Kampen's theorem,
fundamental theorem of algebra, Brouwer fixed point theorem, simplicial
complexes, singular homology, homotopy invariance, long exact sequence
of a pair, excision, Mayer-Vietoris sequence, Euler characteristic, cell
complexes

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1. A continuous map \( f: X \to Y \) induces maps \( f_\#: C_* X \to C_* Y \) and \( f_*: H_* X \to H_* Y \) for singular chains and singular homology. Given two homotopic maps \( f \simeq g: X \to Y \) then the homology maps \( f_* \) and \( g_* \) agree. One proves this fact by constructing a map \( P : C_n X \to C_{n+1} Y \) for each \( n \) such that

\[
Pd + dP = f_\# - g_\#
\]

where \( d \) denotes the boundary map for \( C_* X \) or \( C_* Y \). It suffices to construct the map \( P \) for the special case of \( Y = \Delta^n \), the \( n \) simplex. Explain how to construct \( P \) for that special case.

**Remark:** You do NOT have to verify the property \( Pd + dP = f_\# - g_\# \) or explain how this property forces \( f_* = g_* \). Just explain how to construct \( P \).

2. (a) Define what is meant by an adjunction space.

(b) Give a description of the torus \( T = S^1 \times S^1 \) as an adjunction space.

3. (a) Define what is meant by a vector field on the \( n \) sphere \( S^n \).

(b) Explain why \( S^n \) admits an everywhere non zero vector field only for \( n \) odd.

**Remark:** In proving this fact you may assume any result proved during the course concerning the degrees of maps \( f : S^n \to S^n \).

4. Given a topological space \( X \) and a subspace \( \iota : A \subset X \) suppose that we have the following commutative diagram with exact rows:

\[
\begin{array}{cccccccc}
\cdots & \to & H_3 (A) & \overset{\iota_*}{\longrightarrow} & H_3 (X) & \to & H_3 (X/A) & \overset{\iota_*}{\longrightarrow} & H_3 (X) & \to & \cdots \\
& \parallel & \parallel & \parallel & \parallel & \parallel & \parallel & \parallel & \parallel & \parallel \\
\cdots & \to & \mathbb{Z} & \times 2 \mathbb{Z} & \to & ? & \to & \mathbb{Z}/2\mathbb{Z} & \to & 0 & \to & \cdots 
\end{array}
\]

What are the possibilities for \( H_3 (X/A) \)?

5. Given a cell complex \( X \) explain how one defines the cellular homology of \( X \).

6. (a) State the Lefschetz Fixed Point Theorem.

(b) Let \( P^{2n} \) denote the \( 2n^{th} \) projective plane. You can assume that \( P^{2n} \) is a finite simplicial complex. Show that every continuous map \( f : P^{2n} \to P^{2n} \) has a fixed point.

**Remark:** You can assume any result proved about the homology of \( P^{2n} \).

7. (a) State the Hopf Trace Theorem.

(b) Prove the Hopf Trace Theorem.

**Remark:** In proving the theorem you may assume the result that when you are given an abelian group \( G \) of finite rank, a subgroup \( H \subset G \), and a homomorphism \( \phi : G \to G \) where \( \phi(H) \subset H \) then

\[
\text{Tr} \{ \phi : G \to G \} = \text{Tr} \{ \phi : H \to H \} + \text{Tr} \{ \phi : G/H \to G/H \}.
\]
<table>
<thead>
<tr>
<th>Instructor Name</th>
<th>Teaching Faculty</th>
<th>Teaching Department</th>
<th>Subject</th>
<th>Course Number</th>
<th>Section</th>
<th>Year</th>
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<th>Enrolment</th>
<th>Responses</th>
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1.4.43 Mathematics 4153

Mathematics 4153A/B - Algebraic Geometry

Affine and projective varieties, coordinate rings and function fields, birational correspondences, sheaves, dimension theory, regularity.

Antirequisite(s):
Prerequisite(s): [Mathematics 4120A/B; Mathematics 3154A/B](#) is recommended but not required.

Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
1.4.44 Mathematics 4154

Mathematics 4154A/B - Introduction to Functional Analysis
Banach and Hilbert spaces, dual spaces, annihilators, Hahn-Banach theorem, Riesz representation theorems, bounded linear operators, adjoints, closed graph and Banach-Steinhaus theorems, compact operators, the Fredholm alternative, the operational calculus, spectral resolution of compact normal operators, applications to integral equations.

Antirequisite(s):

Prerequisite(s): Mathematics 2120A/B (or the former 203b), Mathematics 3122A/B, 3124A/B.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
The University of Western Ontario
Department of Mathematics

Mathematics 4154A/9054A, Fall 2008:
Introduction to Functional Analysis

Time and room: to be arranged.

Instructor: Tatyana Foth (tfoth@uwo.ca)

Web page: go to http://www.math.uwo.ca/~tfoth/


Prerequisites:
Mathematics 2120A/B (or the former 203b), 3122A/B, 3124A/B.
Note: old numbers are, respectively, 200a/b, 304a/b, 307a/b.

Course outline:
Banach and Hilbert spaces, dual spaces, annihilators, Hahn-Banach theorem, Riesz representation theorems, bounded linear operators, adjoints, closed graph and Banach-Steinhaus theorems, compact operators, the Fredholm alternative, the operational calculus, spectral resolution of compact normal operators, applications to integral equations, other topics if time permits.

Evaluation of student performance:
1. Tests and written assignments 60%
2. Final examination 40%

The 2.5 hour final examination will be given during the Midyear Examination period in December and will be scheduled by the participants.
What is expected of the student: students are expected to attend all lectures and complete all tests, assignments and examinations.

FOR 4154A STUDENTS:

If tests or assignments are missed due to a medical excuse and involve no more than 10% of the course mark, then the weight of missed coursework will be moved to the 60% tests and assignments component of the mark (it will be rescaled accordingly). For course work missed due to a medical excuse and involving 10% or more of the course mark, see the standard policy below.

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean’s office as soon as possible and contact your instructor immediately. It is the student’s responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean’s Office immediately. For further information please see:


A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record’s Release Form (located in the Dean’s Office) for visits to Student Health Services. The form can be found here:

https://studentservices.uwo.ca/secure/medical_document.pdf
In Problems 1,2 fully explain your answers. In Problems 3-29 clearly indicate whether the statement is true or false, the correct answer is worth 1 mark, no partial credit will be given, you do not have to show your work or explain your answer.

Problem 1. (7 marks)
Let $H = L^2(0,1)$, with the standard inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$.
Define a mapping $A : H \to H$ by
\[ A : f(t) \mapsto (Af)(t) = \begin{cases} 0 & 0 < t \leq \frac{1}{3} \\ f(t) & \frac{1}{3} < t < 1 \end{cases} \]

(a) Show that $A$ is a linear operator on $H$.
(b) Show that $A$ is bounded and find $||A||$.
(c) Is $A$ invertible? Justify your answer.

Problem 2. (6 marks)
(a) Show that the operator $A = SS^* \in \mathcal{L}(l^2)$ is positive.
(b) Find $B \in \mathcal{L}(l^2)$ such that $B^2 = A$.
(c) Is $A$ self-adjoint? Justify your answer.

3. $||x|| = 5|x|$, $x \in \mathbb{R}$, is a norm on $\mathbb{R}$.
   TRUE       FALSE

4. The set $\{x = (x_n)_{n=1}^\infty \in l^2 \mid x_n = 0 \text{ for } n \geq 2\}$ is a linear subspace of $l^2$ which is not closed.
   TRUE       FALSE

5. If $V$ is a linear subspace of a normed space $E$, $V$ is of codimension 1 and not dense in $E$, then $V$ is closed.
   TRUE       FALSE

6. If $y \in l^2$, $y \neq 0$, then for any $x \in l^2$ there is unique $\lambda \in \mathbb{C}$ such that
   \[ ||x - \lambda y|| = \inf_{\mu \in \mathbb{C}} ||x - \mu y|| \]
   TRUE       FALSE

7. Any linear subspace of a Banach space is a Banach space with respect to the induced norm.
   TRUE       FALSE

1
8. If $f$ is a continuous linear functional on a normed space $E$, then the set
\[ \{ x \in E \mid ||x|| = 1, |F(x)| = 1 \} \]
is closed in $E$.

TRUE FALSE

9. If a Hilbert space $H$ contains a complete orthonormal sequence $(x_n)_{n=1}^{\infty}$, then $H$ is isomorphic to $l^2$.

TRUE FALSE

10. If $E = L^2(0, 1)$, with the standard inner product $\langle f, g \rangle = \int_0^1 f(t) \overline{g(t)} dt$ for $f, g \in H$, and $V$ is the 1-dimensional linear subspace of $H$ spanned by $f(t) = t$, then $(V^\perp)^\perp = V$.

TRUE FALSE

11. It is not possible to write $l^2$ as the orthogonal direct sum of two closed non-trivial linear subspaces.

TRUE FALSE

12. The mapping $f : l^1 \to \mathbb{C}$, $x = (x_n)_{n=1}^{\infty} \mapsto x_1 x_2$ defines a continuous linear functional on $l^1$.

TRUE FALSE

13. If $f$ be a continuous linear functional on a normed space $E$, then $|f(x)| = ||f|| ||x||$ for some non-zero $x \in E$.

TRUE FALSE

14. If $E$ is a normed space, $f \in E^*$, then $\ker(f)$ is a closed linear subspace of $E$.

TRUE FALSE

15. If $E = C[0, 1]$ with the sup-norm, then the mapping $E \to \mathbb{C}$, $f(t) \to f(\frac{1}{2})$, defines a continuous linear functional on $E$.

TRUE FALSE

16. Any bounded linear operator on a Banach space is compact.

TRUE FALSE

17. The mapping $SS + S^* : l^2 \to l^2$ is a linear operator on $l^2$.

TRUE FALSE

18. If $H$ is a Hilbert space, $A, B \in \mathcal{L}(H)$ and $A^* = B^*$, then $A = B$.

TRUE FALSE

19. If $H$ is a Hilbert space, $A \in \mathcal{L}(H)$, and $A$ is positive, then
\[ ||A|| = \sup_{||x||=1} |\langle Ax, x \rangle| \]

TRUE FALSE
20. If $A$ is a linear operator on a Banach space $E$ and for any $n \in \mathbb{N}$ there is a non-zero vector $x_n$ in $E$ such that $Ax_n = nx_n$, then $A$ is not continuous.  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE} \\

21. The range of a bounded linear operator on a Hilbert space $H$ is closed in $H$.  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE} \\

22. If $H$ is a Hilbert space, then the set of all invertible bounded linear operators on $H$ is a linear subspace of $\mathcal{L}(H)$.  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE} \\

23. If $H$ is a Banach space, $A \in \mathcal{L}(H)$, and $\|A\| = 2$, then the operator $-3I - A$ is invertible.  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE} \\

24. If $H$ is a Hilbert space, $A \in \mathcal{L}(H)$, $A$ is of finite rank and hermitian, then the set of eigenvalues of $A$ is nonempty.  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE} \\

25. If $H$ is a Hilbert space, $A \in \mathcal{L}(H)$, and $A$ is Hilbert-Schmidt, then $-A$ is Hilbert-Schmidt.  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE} \\

26. The operator $S$ on $l^2$ is a positive operator.  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE} \\

27. If $A$ is a bounded linear operator on a Hilbert space, and $i$ is an eigenvalue of $A$, then $A$ is not positive.  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE} \\

28. If $H$ is a Hilbert space, $A \in \mathcal{L}(H)$ is a contraction, then $2I - A$ is invertible.  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE} \\

29. There is a twice differentiable function $y(x)$ on $[0,1]$ such that  
\begin{align*}
  y''(x) &= \sin(x^2) \\
  y(0) &= y(1) = 0
\end{align*}  
\textbf{TRUE} \hspace{1cm} \textbf{FALSE}
INSTRUCTOR & COURSE EVALUATION
THE UNIVERSITY OF WESTERN ONTARIO

<table>
<thead>
<tr>
<th>Instructor Name</th>
<th>Teaching Faculty</th>
<th>Teaching Department</th>
<th>Subject</th>
<th>Course Number</th>
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1.4.45 Mathematics 4155

Mathematics 4155A/B - Multivariable Calculus

Review of differentiability in Euclidean space, inverse and implicit function theorems, integration in Euclidean space, Fubini’s theorem, partitions of unity, change of variable, multilinear functions, tensor and wedge product, vector fields, differential forms, Poincaré’s lemma, Stokes' theorem, manifolds, fields and forms on manifolds, Stokes' theorem on manifolds.

Antirequisite(s):
Prerequisite(s): Calculus 2503A/B, and Mathematics 3122A/B.
Corequisite(s):
Pre-or Corequisite(s):
Mathematics 4156A/B - Complex Variables II

Moebius transformations; local behavior of analytic functions, open and inverse mapping theorems; Schwarz's lemma; harmonic functions, solution of the Dirichlet problem on the disk, the Jensen and Poisson- Jensen formulas, the Schwarz reflection principle; analytic continuation; normal families, the Riemann mapping theorem, the homotopic version of Cauchy's theorem; conformal mapping.

Antirequisite(s):
Prerequisite(s): Mathematics 3124A/B.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
INSTRUCTOR: André Boivin  
Phone: 519-661-2111, Ext. 86525  
Office: MC 122


PREREQUISITES: Mathematics 3124A/B (old number 307a/b).

COURSE OUTLINE: Conformal mappings, Möbius transformations; the homotopic version of Cauchy's theorem; local behaviour of analytic functions; open and inverse mapping theorems; Goursat's Theorem; Schwarz's lemma; harmonic functions, solution of the Dirichlet problem on the disk; Schwarz reflection principle; analytic continuation; normal families, the Riemann mapping theorem; Weierstrass Factorization Theorem.

This corresponds, approximately, to the following sections in the textbook:
III:3; IV:6,7,8; VI:2; VII:1-5; IX:1,2; X:1,2.

EVALUATION OF STUDENT PERFORMANCE:  
70% Assignments (possibly including in-class presentations)  
30% Final Examination

IMPORTANT SENATE POLICY: Students are responsible for ensuring that their selection of courses is appropriate and accurately recorded and that all course prerequisites have been successfully completed. If the student does not have the prerequisites for a course, and does not have written special permission from his or her Dean to enroll in the course, the student may be removed from the course and it will be deleted from the student’s record. This decision may not be appealed. A student will receive no adjustment to his or her fees in the event that he or she is dropped from a course for failing to have the necessary prerequisites.

STATEMENT ON ACADEMIC OFFENCES: Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site: http://www.uwo.ca/univsec/handbook/appeals/scholoff.pdf
1. Let \( S(z) = \frac{az + b}{cz + d} \) and \( T(z) = \frac{\alpha z + \beta}{\gamma z + \delta} \) be two Möbius transformations. Show that \( S = T \) if and only if there is a nonzero complex number \( \lambda \) such that \( \alpha = \lambda a, \beta = \lambda b, \gamma = \lambda c, \) and \( \delta = \lambda d. \)

5pt

2. Can you map the open unit disc \( \mathbb{D} = \{ z : |z| < 1 \} \) conformally onto the punctured disc \( \{ z : 0 < |z| < 1 \} \)? Justify your answer.

5pt

3. Let \( G \) be an open connected subset of \( \mathbb{C} \), and let \( \{ f_n \}_{n \geq 1} \) be a sequence of holomorphic functions in \( G \) converging uniformly on the compact subsets of \( G \). Let \( f = \lim_{n \to \infty} f_n. \)

(a) Suppose that \( 0 \notin f_n(G) \) for all \( n \geq 1 \). Prove that either \( f \equiv 0 \) or \( 0 \notin f(G). \)

(b) Suppose that \( f_n \) is one-to-one (injective) for all \( n \geq 1 \). Prove that either \( f \) is constant, or \( f \) is one-to-one.

5pt

4. Let \( G \) be an open subset of \( \mathbb{C} \). Show that if \( F \subset \text{Hol}(G) \) is normal, then \( F' = \{ f' : f \in F \} \) is also normal.

5pt

5. State the Riemann mapping theorem.

5pt

6. Let \( \mathbb{D} = \{ z : |z| < 1 \} \) denote the open unit disc, and \( \mathbb{H} = \{ z \in \mathbb{C} : \text{Im}(z) > 0 \} \) denote the upper half-plane. Set

\[
h(z) = i \frac{1 + z}{1 - z}.
\]

Show that \( h \) maps \( \mathbb{D} \) conformally onto \( \mathbb{H} \) and find \( h^{-1}. \)

5pt

7. Let \( \mathbb{D} = \{ z : |z| < 1 \} \) denote the open unit disc and

\[
f(z) = \sum_{n=1}^{\infty} a_n z^n, \quad g(z) = \sum_{n=1}^{\infty} b_n z^n
\]

be two functions holomorphic in \( \mathbb{D} \). Assuming that \( f \) is one-to-one and \( g(D) \subset f(D) \), show that \( |b_1| \leq |a_1|. \)

(Hint: Define \( \ell = f^{-1} \circ g \) and apply Schwarz's Lemma.)

5pt

8. (a) Prove that the infinite product

\[
f(z) = \prod_{n=0}^{\infty} (1 + z^{2^n})
\]

converges uniformly on the compact subsets of the open unit disc \( \mathbb{D} = \{ z : |z| < 1 \}. \)
(b) Show that for all \( z \in D \), we have

\[
f(z) = \frac{1}{1-z}.
\]

(Hint: Find \( P_N \).)

9. (BONUS) Let \( D = \{ z : |z| < 1 \} \) denote the open unit disc, and \( f : \overline{D} \rightarrow \mathbb{C} \) be continuous. Assume that both \( u := \text{Re}(f) \) and \( v := \text{Im}(f) \) are harmonic. Show that \( f \) is analytic on \( D \) if and only if

\[
\int_{-\pi}^{\pi} f(e^{it})e^{int} dt = 0
\]

for all \( n \geq 1 \).

(Hint: Write \( f = u + iv \) and recall that the Poisson kernel is defined as follows:

\[
P_r(\theta) = \sum_{n=-\infty}^{\infty} r^{|n|} e^{in\theta},
\]

for \( 0 \leq r < 1 \) and \(-\infty < \theta < \infty \).)
### Instructor & Course Evaluation

**The University of Western Ontario**

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| 16. Course As Learning Experience | 7 | 0 | 0 | 0 | 0 | 5 | 2 | 6.29  | 0.49              | 6      |

### Mathematics 4157

Mathematics 4157A/B - Complex Variables III

Entire and meromorphic functions, infinite products, canonical products, the Weierstrass factorization and Mittag-Leffler theorems, the Hadamard factorization theorem; simply periodic and doubly periodic functions, elliptic functions; the Picard theorems (with Schottky's, Montel's, and Landau's theorems); the prime number theorem (with the Gamma and Riemann Zeta functions).

**Antirequisite(s):**

**Prerequisite(s):** Mathematics 4156A/B or Mathematics 3124A/B with the permission of the Department.

**Corequisite(s):**
Mathematics 4158A/B/Y - Foundations of Mathematics

Set theory: axioms, ordinal numbers, transfinite induction, cardinality, the axiom of choice.

Foundations of mathematics: construction of the real numbers from the natural numbers by one of the standard methods. First-order logic: propositional calculus, quantifiers, truth and satisfaction, models of first-order theories, consistency, completeness and compactness.

Antirequisite(s):

Prerequisite(s): The permission of the Department.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.
Mathematics 4158A/9058A

Foundations of Mathematics

September 2008

- **Instructor:**
  - Mike Dawes.
  - Office: MC 114.
  - E-mail: mdawes[at]uwo.ca


- **Prerequisites:** Permission of the Department.

- **Lectures:** Tuesdays 1:30pm–3:30pm and Thursdays 1:30pm–2:30pm, Middlesex College 108 both days. First lecture: Tuesday, September 9.

- **Course Outline:** An introductory survey of mathematical logic.
  - Propositional calculus;
  - Predicate Calculus;
  - Completeness—Gödel's theorem;
  - Model Theory;
  - Incompleteness and Undecidability—Gödel, Tarski, Church.

Topics may be added or deleted in accordance with time available and the background of students.

- **Methods of Evaluation:** Problems will be assigned weekly. Marks for the course will be based on written solutions of assigned problems.

  *Note: in courses with no final exam, marks are required to be submitted to the Registrar one week after the last day of classes, i.e. on December 10, one week after December 3. The final due date for assignments (and any revisions) will be December 5."

- **Statement on Academic Offences**

  Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following web site: [http://www.uwo.ca/univsec/handbook/appeals/scholastic_discipline_undergrad.pdf](http://www.uwo.ca/univsec/handbook/appeals/scholastic_discipline_undergrad.pdf).

- **Medical Accommodation**

  - For UWO Policy on Accommodation for Medical Illness see: [http://www.uwo.ca/univsec/handbook/appeals/accommodation_medical.pdf](http://www.uwo.ca/univsec/handbook/appeals/accommodation_medical.pdf)
  - Downloadable Student Medical Certificate (SMC): [https://studentservices.uwo.ca](https://studentservices.uwo.ca) under the Medical Documentation heading
  - Students seeking academic accommodation on medical grounds for any missed tests, exams, participation components and/or assignments worth 10% or more of their final grade must apply to the Academic Counselling office of their home Faculty and provide documentation. Academic accommodation cannot be granted by the instructor or department.
1.49 Mathematics 4160

Mathematics 4160A/B - Ordinary Differential Equations

Laplace transforms and their application to solving differential equations. Sturm-Liouville systems; eigenvalue problems, expansions, Fourier series, autonomous systems; linear and non-linear problems, types of critical points, stability.

Antirequisite(s):
Prerequisite(s): Mathematics 3123A/B; or Applied Mathematics 2402A or the former Differential Equations 2402A.

Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

1.50 Mathematics 4161

Mathematics 4161A/B - Linear Ordinary Differential Equations

First order vector systems and nth order single equations; adjoint systems and boundary value problems; Green's functions and self adjoint eigenvalue problems; expansion theory and spectral decomposition.

Antirequisite(s):
Prerequisite(s): Mathematics 3123A/B; or Applied Mathematics 2402A or the former Differential Equations 2402A.

Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.

1.51 Mathematics 4958

Mathematics 4958A/B - Special Topics in Mathematics

Antirequisite(s):
Prerequisite(s): Permission of the Department.
Corequisite(s):
Pre-or Corequisite(s):
Extra Information: 3 lecture hours, 0.5 course.
1.4.52 Mathematics 4959

Mathematics 4959A/B - Special Topics in Mathematics

Antirequisite(s):

Prerequisite(s): Permission of the Department.

Corequisite(s):

Pre-or Corequisite(s):

Extra Information: 3 lecture hours, 0.5 course.

INSTRUCTOR & COURSE EVALUATION
THE UNIVERSITY OF WESTERN ONTARIO

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