MATRIX TRANSFORMATIONS OF POWER SERIES*

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Abstract. We consider the sequence of transforms (g_n) of a power series $\sum_{n=0}^{\infty} a_n z^n$ given by $g_n(z) := \sum_{k=0}^{\infty} b_{nk} a_k z^k$. We establish necessary and sufficient conditions on the matrix (b_{nk}) for the sequence (g_n) to converge uniformly on compact subsets of the disk $D_P := \{z : |z| < P\}$ to a function holomorphic on D_P.

1. Introduction. Suppose throughout that $0 < P \le \infty$, $0 < R < \infty$, and that all sequences and matrices are complex with indices running through $0, 1, 2, \cdots$. We make the following definitions:

D_P is the disk $\{z : |z| < P\}$;

$\mathcal{E}$ is the set of all sequences $\mathbf{a} \equiv (a_n)$ such that $\lim |a_n|^{\frac{1}{n+1}} = 0$;

$\mathcal{E}^\beta$ is the set of all sequences $\mathbf{a} \equiv (a_n)$ such that $\limsup |a_n|^{\frac{1}{n+1}} < \infty$;

$\mathcal{E}_R$ is the set of all sequences $\mathbf{a} \equiv (a_n)$ such that $\displaystyle\sum_{n=0}^{\infty} |a_n| R^n < \infty$;

$\mathbf{A}_R$ is the set of all sequences $\mathbf{a} \equiv (a_n)$ such that $\limsup |a_n|^{\frac{1}{n+1}} = \dfrac{1}{R}$;

It will follow from the lemma (below) that $\mathcal{E}^\beta$ is the $\beta-$dual of $\mathcal{E}$. The following are the first three of eight theorems we shall prove concerning matrix transformations of power series.

Theorem 1. *A matrix $\mathbf{B} \equiv (b_{nk})$ has the property that whenever the sequence $\mathbf{a} \equiv (a_n) \in \mathcal{E}_R$ the sequence of functions (g_n) given by*

$$g_n(z) := \sum_{k=0}^{\infty} b_{nk} a_k z^k, \;\; n = 0, 1, \cdots,$$

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converges uniformly on every compact subset of D_P*, each power series* $\sum_{k=0}^{\infty} b_{nk} a_k z^k$ *being convergent on* D_P*, if and only if*

(i) $\lim_{n\to\infty} b_{nk} =: b_k$ *for* $k = 0, 1, \cdots$;

(ii) $\sup_{n\geq 0, k\geq 0} |b_{nk}| \big(\frac{p}{R}\big)^k < \infty$ *for each positive* $p < P$.

And then $\lim_{n\to\infty} g_n(z) = \sum_{k=0}^{\infty} b_k a_k z^k$ *on* D_P.