Errata: Simplicial Homotopy Theory

P.G. Goerss and J.F. Jardine

October 13, 2000

The last line of p.148 asserts that "the component of $d^*(x_\sigma)$ which corresponds to the identity on $\mathbf{k}$ is x_σ". This is false, because there could be several surjective ordinal number maps $\gamma : \mathbf{n} \to \mathbf{k}$ such that $\gamma \cdot d = 1_k$ for a fixed ordinal number monic $d : \mathbf{k} \to \mathbf{n}$.

The idea is to show that the kernel of the abelian group homomorphism

$$\Psi : \bigoplus_{\mathbf{n} \to \mathbf{k}} NA_k \to A_n$$

is zero, by induction on n. The direct sum is indexed on all surjective ordinal number morphisms $\sigma : \mathbf{n} \to \mathbf{k}$. Every such σ has a specific section $d_\sigma : \mathbf{k} \to \mathbf{n}$, which is defined by

$$d_\sigma(i) = max\{\ j \mid \sigma(j) = i\ \}$$

The we say, for the ordinal number epis $\sigma, \tau : \mathbf{n} \to \mathbf{k}$, that $\sigma \leq \tau$ if $d_\sigma(i) \leq d_\tau(i)$ for all i. Then one sees that $\sigma \leq \tau$ if $\tau \cdot d_\sigma = 1_{\mathbf{k}}$.

Now suppose that $(x_\sigma) \in \ker \Psi$, and concentrate on the components x_σ corresponding to epimorphisms $\sigma : \mathbf{n} \to \mathbf{k}$, where $k < n$ is fixed. If some $x_\tau \neq 0$, there will be a maximal such τ with respect to the ordering above. But then the component corresponding to $1_{\mathbf{k}}$ of $d_\tau(x_\sigma)$ is of the form $\sum_{\sigma \geq \tau} x_\sigma = x_\tau$ by the maximality assumption. Thus, $\Psi d_\tau(x_\sigma) = 0$ and so $d_\tau(x_\sigma) = 0$, whence $x_\tau = 0$. This is a contradiction, so that $x_\sigma = 0$ for all $\sigma : \mathbf{n} \to \mathbf{k}$ with $k < n$. The remaining component is $x_{1_{\mathbf{n}}} \in NA_n$, and the restriction of Ψ to NA_n is the inclusion $NA_n \hookrightarrow A_n$.

References

[1] P.G. Goerss and J.F. Jardine, *Simplicial Homotopy Theory*, Progress in Mathematics 174, Birkhäuser, Basel-Boston-Berlin (1999).