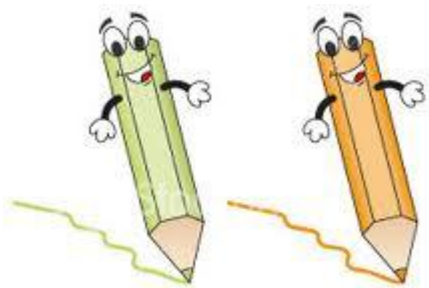
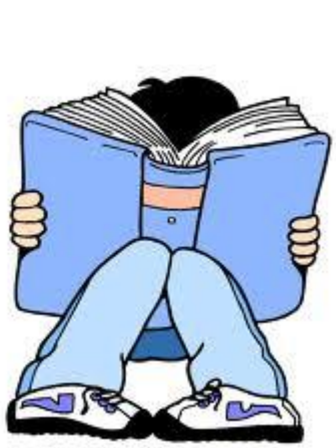
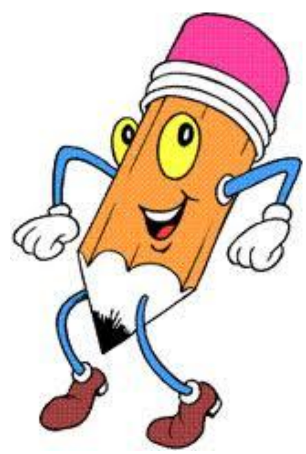
۞ HOMEWORK 1 ۞

☺ *THE JOY OF DISCOVERY* ☺

Mathematics 1120b – Fundamental Concepts in Mathematics





HOMEWORK --- (Due by Monday, January 30, 2012)

Exercise 1. Let $n \in \mathbb{N}$.

Set $A(n) := \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n}$.

a) Evaluate $A(1), A(2), A(3)$ and $A(4)$ case by case, by direct calculations.

b) Determine values $A(n)$ for all $n \in \mathbb{N}$ in the simplest possible terms, and prove that your formula for $A(n)$ is correct.

20 marks

Exercise 2. Prove that for $k, n \in \mathbb{N}$ and $1 \le k \le n$, we have

$$\frac{n+1}{k+1}\binom{n}{k} = \binom{n+1}{k+1}.$$

10 marks

Exercise 3. Let $n \in \mathbb{N}$ and set $B(n) := \binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + \frac{1}{n+1}\binom{n}{n}$.

a) Determine $B(1), B(2), B(3), B(4)$ and $B(5)$ by direct calculation as a fraction $\frac{a}{b}$, where $a, b \in \mathbb{N}$ and a, b have no common factor except 1.

b) Determine $B(n)$ for all n in the simplest possible way, and prove that your result is correct.

(HINT: Multiply $B(n)$ by $n+1$ and use Exercise 2.)

10 marks

Exercise 4. Show that each natural $n \in \mathbb{N}$ can be written as $n = 2k, k \in \mathbb{N}$,

or as $n = 2k + 1, k \in \mathbb{N}$.

30 marks

Exercise 5a. Show that each natural number $n \in \mathbb{N}$ can be written as $n = 4k + s$, where $k \in \mathbb{N}$ or $k = 0$ and $s \in \{0, 1, 2, 3\}$.

Exercise 5b. Show that each square n^2 of a natural number, can be written as $4l + t$, where $l = 0$ or $l \in \mathbb{N}$, and t is either 0 or 1.

(HINT: Use Exercise 5a.)

30 marks