ALGEBRA SEMINAR TALK

WITH

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FRIDAY, DECEMBER 9, 2011

2:30 P.M. – Middlesex College Room 107

"Quotients of algebraic group actions"

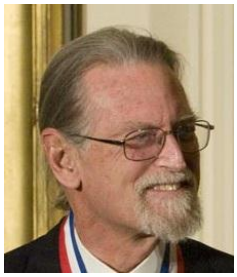
Let $G \times X \to X$ be an action of the algebraic group G on the affine, algebraic variety X. There are two quite different notions of quotient associated with this situation.

If we embrace the conventional approach, we just accept the object $Y = \mathrm{Spec}(k[X]^G)$, along with the natural map $\pi : X \to Y$, as the inevitable thing to study. If G is reductive then this "quotient" is a variety and it has the anticipated universal property, even if Y is not an orbit space. Similar results hold if X is a projective variety. Many important moduli spaces have been constructed using this approach.

But maybe there is another approach, where the emphasis is on orbits of maximal dimension rather than on closed orbits. In this scenario we consider sufficiently small open, G-subsets U of X such that each $G \times U \to U$ has as many desirable properties as the situation will tolerate. If we define U/G by the equation $k[U/G] = k[U]^G$, then we can ask for the following.

(1) $k[U/G]$ is finitely generated.
(2) $k[U/G]$ is a regular ring.
(3) $\pi : U \to U/G$ is surjective.
(4) $\pi : U \to U/G$ separates orbits of maximal dimension.
(5) $\pi : U \to U/G$ has no exceptional divisors.
(6) $\pi : U \to U/G$ is flat.

We do this so as to discard only a small portion of X. We then try to glue all these U/G's together to get a separated quotient variety, without the help of semi-invariants.



D. Mumford

L. Renner

J. Kollar

Western
Science

What a pleasure it is to welcome great, charismatic speakers and born actors like Lex Renner, and various algebraic groups acting on algebraic varieties and making the audience enjoy the drama and freshness of different points of view on quotient spaces, their properties and suspense interspersed with good, light-hearted humour and the pure joy of a happy Friday afternoon. Come enjoy a delightful Algebra Seminar lecture - the final talk in the year 2011!

ALL ARE WELCOME! ☺