



ALGEBRA SEMINAR TALK



WITH

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FRIDAY, JANUARY 13, 2012

2:30 P.M. – Middlesex College Room 107

“The pure virtual braid group is quadratic”

Abstract: If an augmented algebra K over \mathbf{Q} is filtered by powers of its augmentation ideal I , the associated graded algebra $gr_I K$ need not in general be quadratic: although it is generated in degree 1, its relations may not be generated by homogeneous relations of degree 2. In this paper we give a sufficient criterion (called the PVH Criterion) for $gr_I K$ to be quadratic. When K is the group algebra of a group G , quadraticity is known to be equivalent to the existence of a (not necessarily homomorphic) universal finite type invariant for G . Thus the PVH Criterion also implies the existence of a universal finite type invariant for the group G . We apply the PVH Criterion to the group algebra of the pure virtual braid group (also known as the quasi-triangular group), and show that the corresponding associated graded algebra is quadratic, and hence that these groups have a universal finite type invariant.

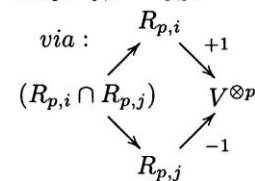


Peter Lee

Koszulness

Let $R_{p,i} := V^{\otimes i} \otimes R \otimes V^{\otimes p-i-2}$

$$\bigoplus_{i < j} (R_{p,i} \cap R_{p,j}) \xrightarrow{d_2} \bigoplus_i R_{p,i} \xrightarrow{d_1} V^{\otimes p} \rightarrow \mathfrak{pvb}_n^p \rightarrow 0$$



For Koszul algebra
([BEER]: e.g. \mathfrak{pvb}_n)
this is exact



Dror Bar-Natan



Leonid Positselski

Miraculously some fundamental objects like Galois cohomology, certain important homogeneous coordinate algebras, some algebras related to configuration spaces of surfaces, and now also $gr_I K$, where K is a group algebra of a pure virtual braid group --- all of these are quadratic. What is going on? How difficult is it to check quadraticity and Koszulness, and how does this reflect on the structure of the basic objects that we are interested in, in the first place? We are lucky that Peter Lee succeeded in obtaining these important results above, in his recent thesis. Thus Peter is able to provide some answers to these tantalizing questions. As some of you may observe, the Koszul algebra hides a beer in the Grad pub which is exactly where we shall proceed to go after this nice lecture, to continue our exciting discussions! Peter’s lecture will be a very exciting beginning of this new year 503 + 503 + 503 + 503 + the new semester. All are very welcome. :-)



ALL ARE WELCOME! ☺