Knot theory and Nature

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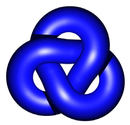
- Polymers routinely become knotted.
- For proteins, knot theory is likely to be useful, not because proteins are or are not knots, but because the geometry and motions of protein backbones may be modelled using techniques from knot theory.
  Two proteins or subsegments of proteins are similar if there is a motion that transforms one into the other while avoiding backbone self-collisions.
  Knot invariants help assess the similarity.
- Measuring changes in crossing number have also been instrumental in understanding the termination of DNA replication and the role of enzymes in recombination.

- To understand the supercoiling and knotting behaviours observed in DNA, in particular DNA packing and unpacking by topoisomerases.
  Wasserman, Cozzarelli and company have predicted that certain enzymes would knot DNA molecules.
  Site specific recombination involves topological changes in the DNA substrate.
- Viruses can knot DNA.
- To characterize the process of gene descrambling in ciliates by forming a short sequence of knots via simple surgeries.
- Chirality of catananes etc. are important to chemists.
- The periodic orbits of the Lorenz attractor define knots.
- Knot invariants appear in String theory.
- All fabrics are knotted and tying your shoes makes a trefoil knot if the ends are glued together.

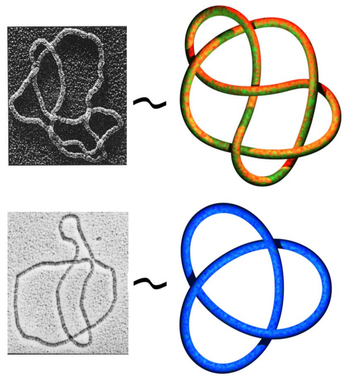


Knotted or Unknotted?

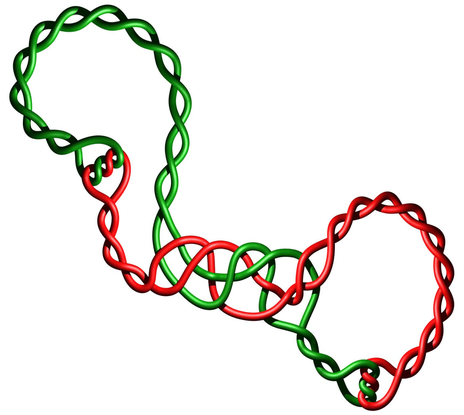
The Unknot

- To determine knottedness is one of the central problems in Knot theory.

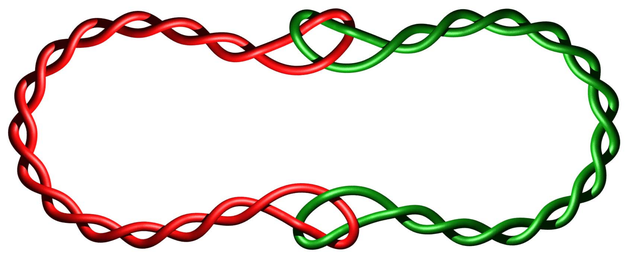
Knotted DNA



Synthetic DNA



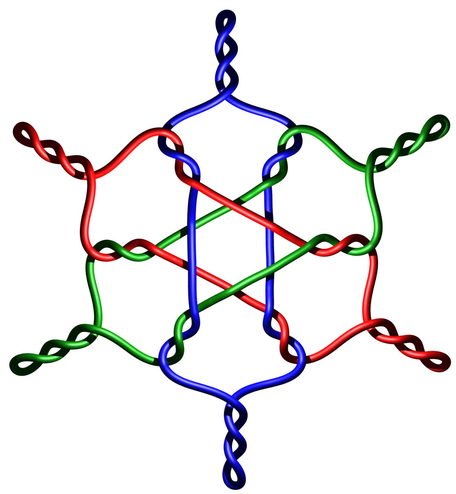
- A minimal (split group 14/13) configuration of the link above.



$1^0_1, -3^0_1, 1^1_1, -27^0_1, -1^0_2, 3^0_2, -1^1_2, -27^0_1 : 1^0_1, -3^0_1, 1^1_1, -27^1_1, -1^1_2, 3^0_2, -1^0_2, -27^1_1$

- The sequence of indexed integers above is a complete invariant for this link called the Master Array.

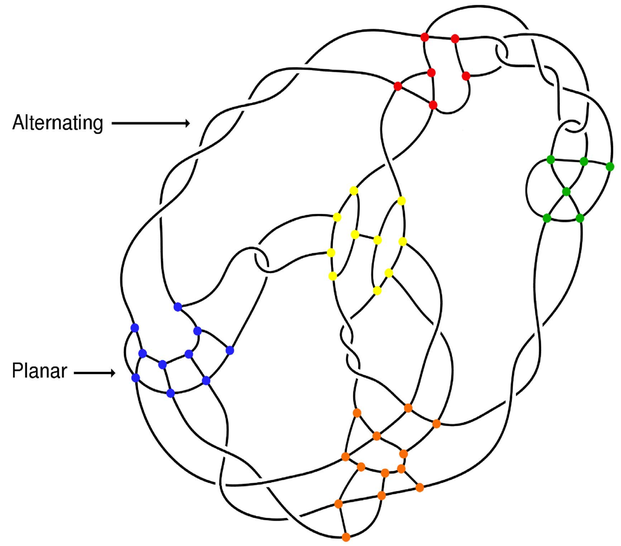
- Topology governs the dynamics and here Nugatory stacks attract.



Knot Theory and Spatial Graphs

- The theory of knots studies the entanglement of loops in space and many biological molecules, such as DNA form loops.
- Other biological molecules form more exotic shapes than simple loops such as proteins which contain extensive cross linking between cystine residues, thus are more complicated structures. In biological systems, proteins are often associated with membranes, so some portions of the molecule are prevented from interacting with others.
- The study of graphs embedded in space is known as spatial graph theory, a natural extension of both knot and graph theory.
- Another application of the theory is to predict possible molecular configurations.

A Planar Alternating Spatial Graph



- Planar alternating spatial graphs may be sufficient to study any molecule of interest.
- Our team can construct a complete invariant for planar alternating spatial graphs in linear time.

The Future of Engineering

- Constructing benign structures with desirable properties.
- Mastering self assembly of lattices.
- A nanotech revolution such as delivery systems for drugs.
- Intelligent design (us) for immune system enhancement.