Calculus 1100 Midterm
Multiple Choice Answers

CODE 111 BEC AAC CEB AEB AEA DCC CD
EDC CB BDCAE

CODE 222 ECB ACA EBC EBA EAA CCD DC
DCE BC BDCAE

CODE 333 CBE CAA BCE BAE AAE CDC CD
CED CB BDCAE

PART B (40 marks)

6 marks B1. Consider the function given by

$$f(x) = \begin{cases} -x+2 & \text{for } x \le 1 \\ x^2 & \text{for } 1 < x \le 2 \\ 5-x & \text{for } x > 2 \end{cases}$$

State the value of the indicated limit, if it exists, in the space provided. If a limit does not exist, write DNE.

(a) $\lim_{x\to 1^+} f(x) =$ $\lim_{x \to 1^+} x^2 = 1$

(b) $\lim_{x\to 1^-} f(x) =$ $\lim_{x \to 1^-} -x+2 = -1+2 = 1$

(c) $\lim_{x\to 1} f(x) =$ 1

(d) $\lim_{x\to 2^+} f(x) =$ $\lim_{x \to 2^+} 5-x = 3$

(e) $\lim_{x\to 2^-} f(x) =$ $\lim_{x \to 2^-} x^2 = 4$

(f) $\lim_{x\to 2} f(x) =$ DNE

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

4 *marks* B2. The function

$$f(x) = \begin{cases} \dfrac{2\sin(x)}{x} & \text{for } x \leq 0 \\ c(x+1) & \text{for } x > 0 \end{cases}$$

is continuous at 0. What is the value of c?

$$\lim_{x \to 0^+} c(x+1) = c \quad (1)$$

$$\lim_{x \to 0^-} 2\frac{(\sin(x))}{x} = 2 \quad (1),$$

since $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

Since f is continuous at 0

$$\therefore \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) \Rightarrow \boxed{c = 2}$$

(1) (1)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B3. The limit $\lim_{h \to 0} \frac{e^{2+h} - e^2}{h}$ is the derivative of some function f at some number a.

(*i*) What are $f(x)$ and a?

Answers: $f(x) =$ e^x ~~(½)~~ (1) +1 if exp

$a =$ 2 ~~(½)~~ (1). means

(*ii*) Compute $\lim_{h \to 0} \frac{e^{2+h} - e^2}{h}$ by evaluating $f'(a)$.

$f'(x) = e^x$ (1)

$f'(a) = e^2$ (1).

correct if wrong ans carried through

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B4. Find all values of x in the interval $[0, \pi]$ that satisfy the equation $\sin(2x) = \sin(x)$.

$\sin(2x) = \sin x$

(1) $2\sin x \cos x - \sin x = 0$

$2\sin x(\cos x - \frac{1}{2}) = 0$

true when (1) $\sin x = 0$ or (1) $\cos x = 1/2$

$\Downarrow$ $x = 0, \pi$ (1)

$\Downarrow$ $x = \pi/3$ (1)

$\therefore \{0, \pi/3, \pi\}$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 *marks* B5. Find $\frac{dy}{dx}$ as a function of x if $y = x^{\ln 2x}$.

$$y = x^{\ln 2x}$$

$$\ln y = \ln 2x \ln x \qquad 1$$

$$y'/y = \frac{\ln 2x}{x} + \frac{\ln x}{\cancel{2}x}(\cancel{2}) \qquad 2$$

$$y' = x^{\ln 2x}\left[\frac{\ln 2x}{x} + \frac{\ln x}{x}\right] \qquad 1$$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEMS ON THIS PAGE.

5 *marks* B6. (a) Suppose the function $y = f(x)$ satisfies the equation $y^7 + 3x^2y^2 + 5x^2 = 9$. Determine $\dfrac{dy}{dx}$ as an expression in x and y.

(2) $7y^6y' + 3x^2 2yy' + 6xy^2 + 10x = 0$

$y'(7y^6 + 3x^2 2y) = -6xy^2 - 10x$

(1) $y' = \dfrac{-6xy^2 - 10x}{7y^6 + 3x^2 2y}$

$= \dfrac{-6xy^2 - 10x}{7y^6 + 6x^2 y}$

(b) Find the equation of the tangent line to the curve $y^7 + 3x^2y^2 + 5x^2 = 9$ at the point $(1, 1)$

$x = 1$, $y = 1$

(1) $y' = \dfrac{-6-10}{7+6} = \dfrac{-16}{13}$

(1) $y - 1 = -\dfrac{16}{13}(x - 1)$

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 marks B7. Use the Intermediate Value Theorem to prove that the polynomial $f(x) = 3x^3 + 2x^2 + 5x - 5$ has a root between 0 and 1.

let $y = 3x^3 + 2x^2 + 5x - 5$ on the closed interval $[0, 1]$

(1) $f(x)$ is continuous because it is a polynomial

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$f(0) = -5 < 0$ (1)

$f(1) = 3 + 2 + 5 - 5 = 5 > 0$ (1)

$\therefore$ by the I.V.T

there exists a number $c \in (0, 1)$

s.t $f(c) = 0$ (1)

$\therefore$ by the I.V.T there is at least 1 root. (1)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.

5 *marks* B8. Find $\dfrac{dy}{dx}$ if $y = \tan^3(e^{\sin x})$. DO NOT SIMPLIFY your answer.

$$\frac{dy}{dy} = 3\tan^2(e^{\sin x})\sec^2(e^{\sin x})\,e^{\sin x}\cos x$$