

CODE III

Calculus 1000A Midterm Examination Friday, October 17, 2008

PART A (50 marks)

**NOTE: YOUR ANSWERS TO THE PROBLEMS ON THIS PAGE MUST BE INDICATED ON THE SCANTRON SHEET.**

- A1. [2 marks] Find the exponential function  $f(x) = Ca^x$ , where  $a > 0$ , whose graph passes through the points with coordinates (0, 3) and (2, 48).

A: $3(3^x)$	B: $2(2^x)$	C: $3(4^x)$	D: $2(4^x)$	E: $2(-4)^x$
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- A2. [2 marks] Find a formula for the inverse function  $f^{-1}(x)$  where  $f(x) = 1 + e^{2x}$ .

A: $2\ln(x-1)$	B: $\frac{1}{2}\ln(x-1)$	C: $2\ln(x+1)$	D: $\frac{1}{2}\ln(x+1)$	E: $\ln(2x-1)$
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- A3. [2 marks] Find the exact value of  $\log_5 10 + \log_5 20 - 3\log_5 2$ .

A: 32	B: 24	C: 2	D: 0	E: 25
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- A4. [2 marks] Find the exact value of  $\arctan \left[ \tan \left( \frac{3\pi}{4} \right) \right]$ .

A: $\frac{3\pi}{4}$	B: $\frac{\pi}{4}$	C: $\frac{5\pi}{4}$	D: $\frac{7\pi}{4}$	E: $-\frac{\pi}{4}$
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- A5. [2 marks] Find the exact value of  $\cos \left[ 2 \arccos \left( \frac{1}{3} \right) \right]$ .

A: $\frac{2}{3}$	B: $\frac{7}{9}$	C: $-\frac{7}{9}$	D: $-\frac{2}{3}$	E: $\frac{2}{9}$
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- A6. [2 marks] Determine  $\lim_{x \rightarrow 1^-} \frac{|x-1|}{x-1}$ .

A: 1	B: -1	C: does not exist	D: 0	E: -2
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- A7. [2 marks] Determine  $\lim_{x \rightarrow -2^-} \frac{10}{x+2}$ .

A: 10	B: -1	C: $\infty$	D: $-\infty$	E: 5
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- A8. [2 marks] If, for all values of  $x$ ,  $\cos(x + \pi) \leq f(x) \leq x^2 - 1$ , find  $\lim_{x \rightarrow 0} f(x)$ .

A: 0	B: 1	C: -1	D: 2	E: does not exist
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- A9. [2 marks] Determine  $\lim_{x \rightarrow -\infty} \tan^{-1}(x^2 - x)$ .

A: $\frac{\pi}{2}$	B: $-\frac{\pi}{2}$	C: $\infty$	D: $-\infty$	E: 0
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A10. [2 marks] Determine  $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2 + 1}}{x - 3}$ .

A: $\infty$	B: $-\infty$	C: 0	D: -3	E: 3
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A11. [2 marks] Determine  $\lim_{x \rightarrow 0^+} \ln(e^x - 1)$ .

A: $\infty$	B: $-\infty$	C: 0	D: 1	E: $e$
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A12. [2 marks] If  $y = \cos(e^x)$ , find  $\frac{dy}{dx}$ .

A: $e^x \cos(e^x)$	B: $e^x \sin(e^x)$	C: $-e^x \sin(e^x)$	D: $\sin(e^x)$	E: $-\sin(e^x)$
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A13. [2 marks] If  $f(x) = e^{\sec x}$ , find  $f'(x)$ .

A: $e^{\sec x}$	B: $e^{\sec x} \tan x$	C: $e^{\sec x} \sec x \tan x$	D: $e^{\sec x} \sec^2 x$	E: $\sec x e^{\sec x - 1}$
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A14. [2 marks] If  $y = \sin(\cos x)$ , find  $\frac{dy}{dx}$ .

A: $\cos(\cos x)$	B: $\sin(-\sin x)$	C: $\sin x \cos(\cos x)$	D: $(-\sin x) \cos(\cos x)$	E: $\cos(-\sin x)$
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A15. [2 marks] If  $y = \arcsin(2x^2)$ , find  $\frac{dy}{dx}$ .

A: $\frac{4x}{\sqrt{1-4x^4}}$	B: $\frac{4x}{\sqrt{1-4x^2}}$	C: $\frac{4x}{\sqrt{1+4x^2}}$	D: $\frac{4x}{1+4x^4}$	E: $8x[\arcsin(2x^2)][\arccos(2x^2)]$
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A16. [2 marks] If  $y = \tan^{-1}(e^{3x})$ , find  $\frac{dy}{dx}$ .

A: $\frac{3e^{3x}}{\sqrt{1+e^{6x}}}$	B: $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$	C: $\frac{3e^{3x}}{1+e^{5x}}$	D: $\frac{e^{3x}}{1+e^{6x}}$	E: $\frac{3e^{3x}}{1+e^{6x}}$
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A17. [2 marks] Suppose  $F(x) = f(g(x))$  and  $f(0) = 1$ ,  $f(6) = 3$ ,  $f'(0) = 2$ ,  $f'(6) = -1$ ,  $g(0) = 6$ ,  $g(6) = 5$ ,  $g'(0) = 2$  and  $g'(6) = 0$ . Find  $F'(0)$ .

A: -2	B: 30	C: -6	D: 18	E: 15
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A18. [2 marks] Suppose  $f(x) = 10^{\tan x}$ . Find  $f'\left(\frac{\pi}{4}\right)$ .

A: $20 \ln 10$	B: $10 \ln 10$	C: 1	D: $\ln 10$	E: 10
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A19. [2 marks] Determine  $\lim_{x \rightarrow 0} \frac{\tan 4x}{x}$ .

A: 0	B: $\frac{1}{4}$	C: 1	D: 4	E: does not exist
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A20. [2 marks] Determine  $\frac{d}{dx}(e^2)$ .

A: $2e$	B: 0	C: $e^2$	D: 1	E: $e$
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A21 - A25. For each of the following, choose the letter which labels the graph below and transfer each of your answers to the scantron sheet.

A21. [2 marks] The graph of  $y = \cos^{-1} x$  is answer on the scantron sheet.

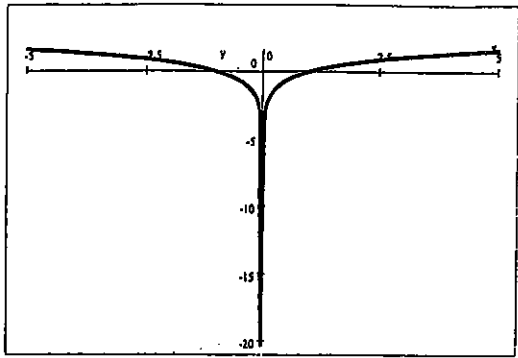
A22. [2 marks] The graph of  $y = \arctan x$  is answer on the scantron sheet.

A23. [2 marks] The graph of  $y = 2^{-x}$  is answer on the scantron sheet.

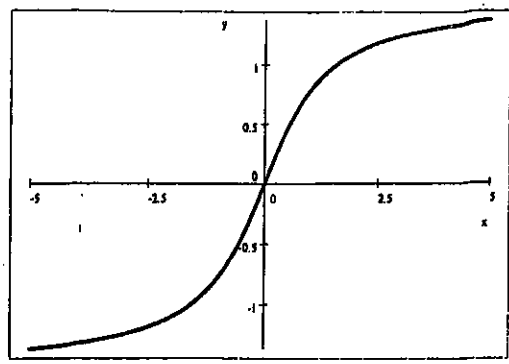
A24. [2 marks] The graph of  $y = \ln |x|$  is answer on the scantron sheet.

A25. [2 marks] The graph of  $y = 1 - e^x$  is answer on the scantron sheet.

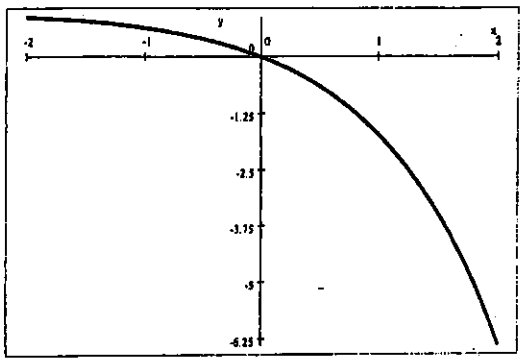
A



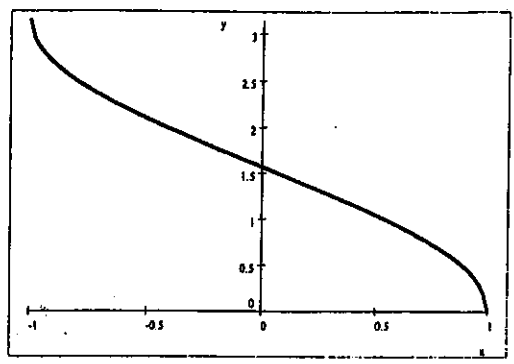
B



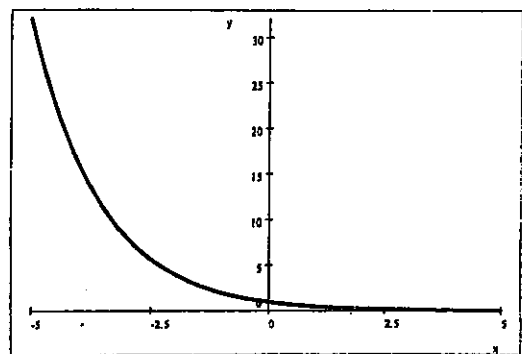
C



D



E



**PART B (50 marks)**

B26. [8 marks] Let  $f(x)$  be the function given by

$$f(x) = \begin{cases} 1 + \ln(-x) & \text{if } x \leq -1 \\ e^{x+1} & \text{if } -1 < x \leq 0 \\ \cos x & \text{if } x > 0 \end{cases}$$

(a) State the value of the indicated limit, if it exists, in the space provided. If a limit does not exist, write DNE.

(i)  $\lim_{x \rightarrow -1^-} f(x)$

(ii)  $\lim_{x \rightarrow -1^+} f(x)$

(iii)  $\lim_{x \rightarrow -1} f(x)$

(iv)  $\lim_{x \rightarrow 0^-} f(x)$

(v)  $\lim_{x \rightarrow 0^+} f(x)$

(vi)  $\lim_{x \rightarrow 0} f(x)$

(b) Is  $f$  continuous at  $-1$ ? Justify your answer.(c) Is  $f$  continuous at  $0$ ? Justify your answer.**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

- B27. [4 marks] For what value of the constant
- $c$
- is the function
- $f$
- , given below, continuous at
- $3$
- ? Justify your answer.

$$f(x) = \begin{cases} 2c - x & \text{if } x < 3 \\ cx^2 - c & \text{if } x \geq 3 \end{cases}$$

**NOTE: SHOW ALL YOUR WORK FOR PART (ii) OF THE PROBLEM ON THIS PAGE.**

- B28. [4 marks] The limit
- $\lim_{x \rightarrow 0} \frac{\arccos(x) - \frac{\pi}{2}}{x}$
- is the derivative of some function
- $f$
- at some number
- $a$
- .

(i) What are  $f(x)$  and  $a$ ? (Answers alone are sufficient.)Answers:  $f(x) =$  $a =$ (ii) Compute (evaluate)  $\lim_{x \rightarrow 0} \frac{\arccos(x) - \frac{\pi}{2}}{x}$  by finding  $f'(x)$  and then evaluating  $f'(a)$ , where  $f$  and  $a$  are the answers you gave in part (i).**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

- B29. [4 marks] Find
- $\frac{dy}{dx}$
- if
- $y = \tan[\sec(x^2)]$
- .
- Do not simplify your answer.

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

- B30. [6 marks] Use the Intermediate Value Theorem to show that there is a root of (i.e., a solution of) the equation
- $\tan 2x = \cos x$
- in the interval
- $(0, \frac{\pi}{6})$
- .

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

- B31. [4 marks] Find
- $\frac{dy}{dx}$
- if
- $y = 3 \sin^2(\sqrt{x^2 + 1})$
- .
- Do not simplify your answer.

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

- B32. [6 marks] Find
- $\frac{dy}{dx}$
- if
- $y = \cos^2[\sin^{-1}(e^x)]$
- .
- Do not simplify your answer.

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- B33. [6 marks] Determine
- $\lim_{x \rightarrow 0} \ln[\cos^{-1}(1 - x^2)]$
- .

**NOTE: SHOW ALL YOUR WORK FOR THE PROBLEM ON THIS PAGE.**

- B34. [8 marks] Find an equation of the tangent line to the graph of

$$\tan^{-1} \left( \frac{2x}{y} \right) = \frac{\pi x}{y^2}$$

at the point  $(1, 2)$ .