Review of trigonometry

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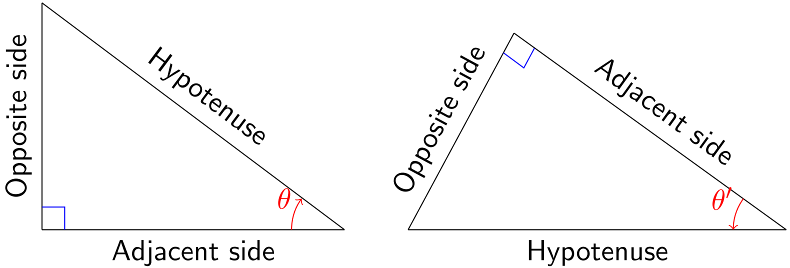
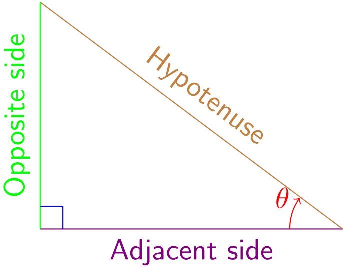
Right triangles



A right triangle, the same after some rotation and symetry. Note that the angle is mesured from the adjacent side toward the hypothenuse and that (here) θ' is the opposite of θ and can be chosen between 0 and $\pi/2$ (whereas θ can be chosen between $-\pi/2$ and 0).



If θ denote the angle in radian and opposite, adjacent and hypothenuse denote the respective length of the corresponding sides, we have:

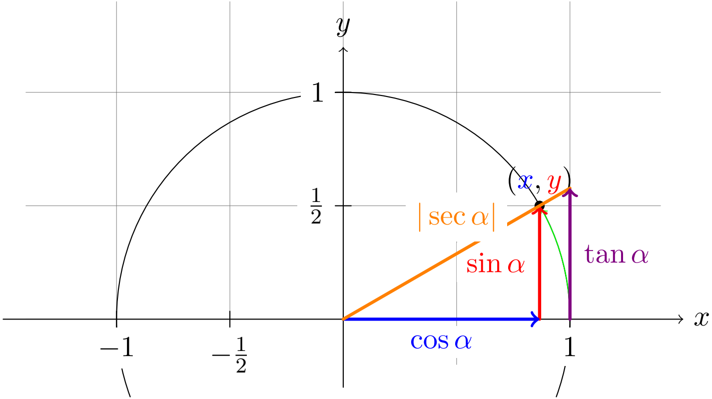
$$|\sin\theta| = \frac{\text{opposite}}{\text{hypothenuse}} \qquad |\cos\theta| = \frac{\text{adjacent}}{\text{hypothenuse}}$$

$$|\tan\theta| = \frac{\text{opposite}}{\text{adjacent}} \qquad |\cot\theta| = \frac{\text{adjacent}}{\text{opposite}}$$

$$|\sec\theta| = \frac{\text{hypothenuse}}{\text{adjacent}} \qquad |\csc\theta| = \frac{\text{hypothenuse}}{\text{opposite}}$$

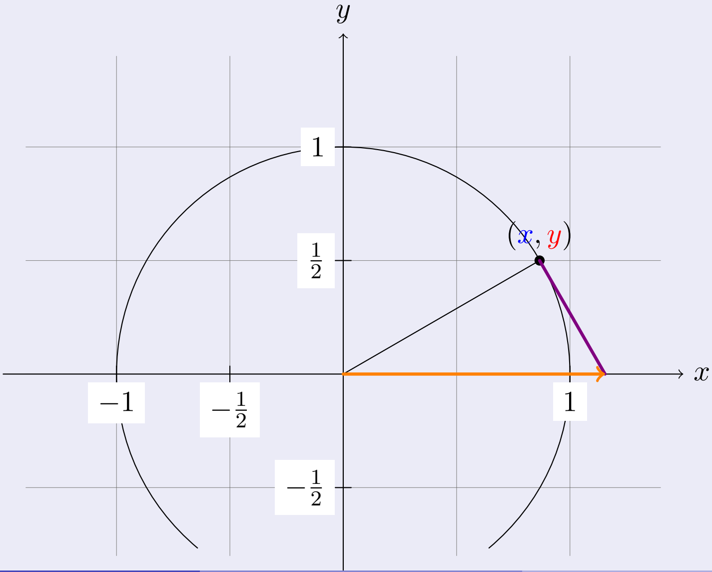
Trigonometric functions and the trigonometric circle

Take a circle with center the origin and radius 1. We can recover our trigonometric functions (and their signs !) from the circle.
Let (x, y) be the coordinatesof a point on our circle



Exercise

What are the values of the orange and violet length ?



Definition

In the trigonometric circle we have

$$\begin{array}{cc} \cos\alpha = x & \sin\alpha = y \\ \tan\alpha = y/x & \cot\alpha = x/y \\ \sec\alpha = 1/x & \csc\alpha = 1/y \end{array}$$

(This gives us not only the absolute value of our trigonometric functions but also their signs.)

Thus ...

$$\sec\alpha = \frac{1}{\cos\alpha} \qquad \csc\alpha = \frac{1}{\sin\alpha}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha}$$

Radian, degrees

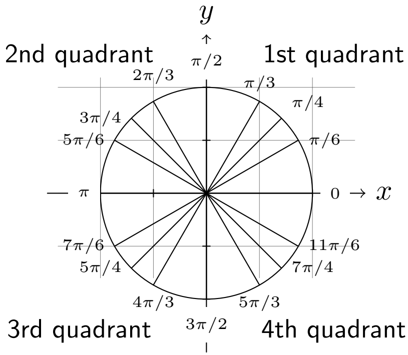
Definition

The value of an angle mesured in radians is $2\pi/360$ times its value in degrees.
The value of an angle mesured in degrees is $\frac{360}{2\pi}$ times its value in degrees.

For instance, you should be familiar with the following values :

360^o	is the same as	2π	radians.
180^o	is the same as	π	radians.
90^o	is the same as	$\pi/2$	radians.
45^o	is the same as	$\pi/4$	radians.
30^o	is the same as	$\pi/6$	radians.
60^o	is the same as	$\pi/3$	radians.

Some angles and trigonometric values

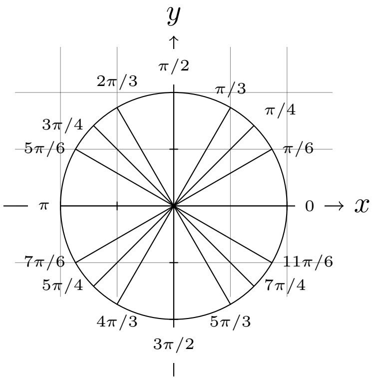


Degrees	Radians	Sine	Cosine
0	0	0	1
30	$\pi/6$	$1/2$	$\sqrt{3}/2$
45	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$
60	$\pi/3$	$\sqrt{3}/2$	$1/2$
90	$\pi/2$	1	0

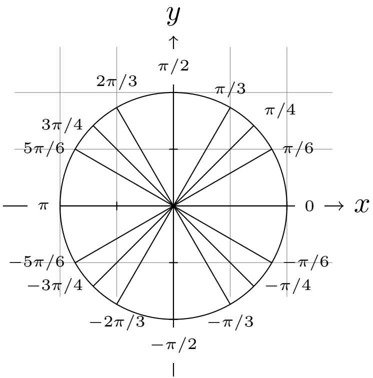
(Recall that π is an irrational number and is about
3.14 159 265358979323846 26433832795028841971693993751
058209749445923078164062862089986280348253421170679
82148086513282306647093844609550582231725359 4 081284
811174502841027019385211055596446229489549303819644 2
881097566593344612847564823378678316527120190914564 8
566923460348610454326648213393607 26. . .)

Two ways of naming the oriented angles

In $[0, 2\pi)$ or



in $(-\pi, \pi]$.



Exercise

Find all values of x in $[0, 2\pi)$ that satisfy the equation $\sec^2 x = 4$.

(Source:*midterm examination 2003.*)

Easy identities

The functions $\alpha \mapsto (\cos \alpha)^2$ and $\alpha \mapsto (\sin \alpha)^2$ are often denoted respectively $\alpha \mapsto \cos^2 \alpha$ and $\alpha \mapsto \sin^2 \alpha$.

From Pythagoras

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

In particular, a sine or a cosine is a number between -1 and 1.

Symetries and period

$$\begin{array}{rcrrcr}
\sin(-\alpha) & = & -\sin\alpha & \cos(-\alpha) & = & \cos\alpha \\
\sin(\alpha + \pi/2) & = & \cos\alpha & \cos(\alpha + \pi/2) & = & -\sin\alpha \\
\sin(\alpha + 2\pi) & = & \sin\alpha & \cos(\alpha + 2\pi) & = & \cos\alpha
\end{array}$$

Addition and multiplication

Additive formulæ

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

In particular $\sin 2\alpha = 2\sin\alpha\cos\alpha$ and
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1$.
There are also formulæ for $\tan$ and $\sec$ ...

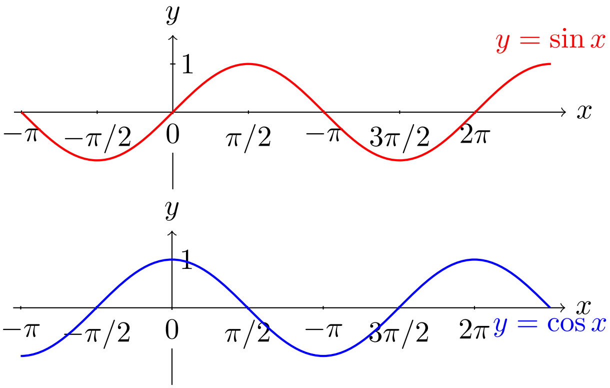
Multiplicative formulæ

$$\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$$

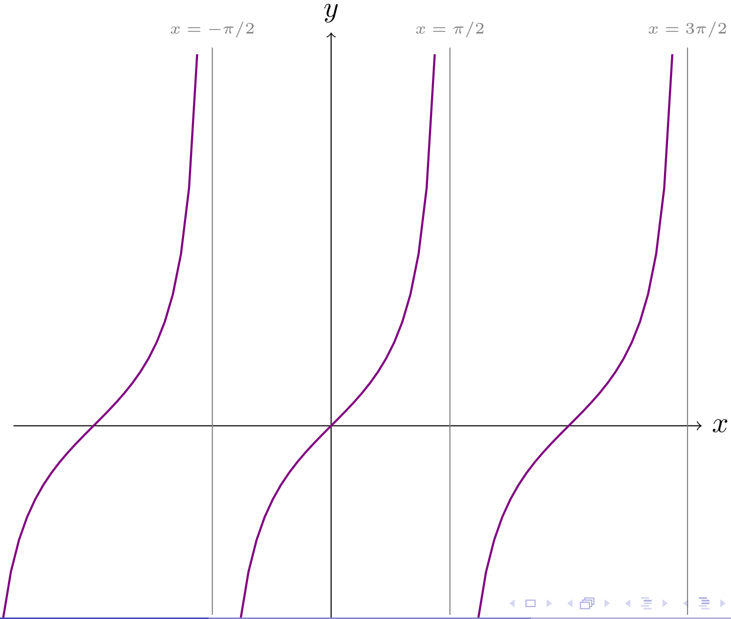
$$\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$

Graphs

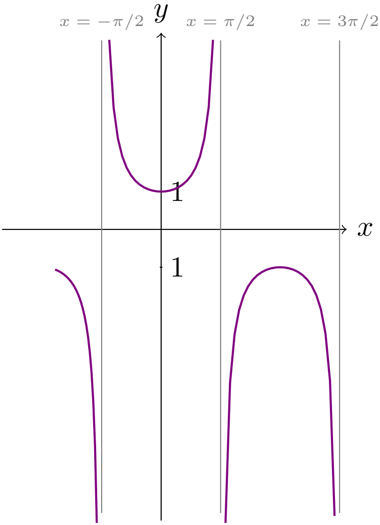


Both the functions sine and cosine have period 2π, domain $\mathbb{R}$ and range $[-1, 1]$.
Sine is an **odd** function.
Cosine is an **even** function,

The tangent function is odd and π-periodic



The graph of secant



Trigonometric functions and derivation

The derivation rules for trigonometric functions are :

f	f'
$x \mapsto \cos x$	$x \mapsto -\sin x$
$x \mapsto \sin x$	$x \mapsto \cos x$
$x \mapsto \tan x$	$x \mapsto \sec^2 x$
$x \mapsto \cot x$	$x \mapsto -\csc^2 x$
$x \mapsto \sec x$	$x \mapsto \sec x \tan x$
$x \mapsto \csc x$	$x \mapsto \csc x \cot x$