

**Problem Set 10**  
November 27, 2023

All numbered exercises are from the textbook *Calculus Vol. 3*, by OpenStax.

1. Exercises 4.8.359–385 (odd only).
2. Find the extreme values of  $f$  subject to both constraints:
  - (a)  $f(x, y, z) = x + y + z$ ;  $x^2 + z^2 = 2$ ,  $x + y = 1$
  - (b)  $f(x, y, z) = z$ ;  $x^2 + y^2 = z^2$ ,  $x + y + z = 24$
  - (c)  $f(x, y, z) = x^2 + y^2 + z^2$ ;  $x - y = 1$ ,  $y^2 - z^2 = 1$ .
3. Find the extreme values of  $f$  on the region described by the inequality:
  - (a)  $f(x, y) = x^2 + y^2 + 4x - 4y$ ;  $x^2 + y^2 \leq 9$
  - (b)  $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ ;  $x^2 + y^2 \leq 16$
  - (c)  $f(x, y) = e^{-xy}$ ;  $x^2 + 4y^2 \leq 1$ .
4. Use Lagrange multipliers to prove that the rectangle with maximum area that has a given perimeter  $p$  is a square.
5. The plane  $x + y + 2z = 2$  intersects the paraboloid  $z = x^2 + y^2$  in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.
6. The plane  $4x - 3y + 8z = 5$  intersects the cone  $z^2 = x^2 + y^2$  in an ellipse. Find the highest and lowest points on the ellipse.