Problem Set 3

October 2, 2023

All numbered exercises are from the textbook *Calculus Vol. 3*, by OpenStax.

1. Exercises 3.1.3–25 (odd only).

2. Exercise 3.1.34.

3. Exercises 3.2.41–61 (odd only).

4. Exercise 3.2.60.

5. Find parametric equations for the tangent line to the curve defined by $\mathbf{r}(t)$ at the specified point.

(a) $\mathbf{r}(t) = \left\langle t^2 + 1, 4\sqrt{t}, e^{t^2 - t} \right\rangle$, $(2, 4, 1)$

(b) $\mathbf{r}(t) = \langle \ln(t+1), t\cos(2t), 2^t \rangle$, $(0, 0, 1)$

(c) $\mathbf{r}(t) = \langle e^{-t}\cos t, e^{-t}\sin t, e^{-t} \rangle$, $(1, 0, 1)$

(d) $\mathbf{r}(t) = \left\langle \sqrt{t^2 + 3}, \ln(t^2 + 3), t \right\rangle$, $(2, \ln 4, 1)$

6. If the curve has the property that the position vector $\mathbf{r}(t)$ is always perpendicular to the tangent vector $\mathbf{r}'(t)$, show that the curve lies on a sphere centered at the origin.

7. If $\mathbf{u}(t) = \mathbf{r}(t) \cdot [\mathbf{r}'(t) \times \mathbf{r}''(t)]$, show that

$$\mathbf{u}'(t) = \mathbf{r}(t) \cdot [\mathbf{r}'(t) \times \mathbf{r}'''(t)] .$$