Practice Term Test 1

1. True or False:

   (i) For any two vectors $\mathbf{u}, \mathbf{v} \in V_3$, we have $\|\mathbf{u} \cdot \mathbf{v}\| = \|\mathbf{u}\| \, \|\mathbf{v}\|$.

   A: YES B: NO

   (ii) For any two vectors $\mathbf{u}, \mathbf{v} \in V_3$, we have $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$.

   A: YES B: NO

   (iii) For any two vectors $\mathbf{u}, \mathbf{v} \in V_3$, we have $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{v} \times \mathbf{u}\|$.

   A: YES B: NO

   (iv) For any two vectors $\mathbf{u}, \mathbf{v} \in V_3$, we have $(\mathbf{u} + \mathbf{v}) \times \mathbf{v} = \mathbf{u} \times \mathbf{v}$.

   A: YES B: NO

   (v) Given $\mathbf{u}, \mathbf{v} \in V_3$, if $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

   A: YES B: NO

   (vi) The vector $\langle 6, -2, 14 \rangle$ is parallel to the plane $3x - y + 7z = 1$.

   A: YES B: NO

   (vii) The curve with vector equation $\mathbf{r}(t) = t^5\mathbf{i} - 3t^5\mathbf{j} + 4t^5\mathbf{k}$ is a straight line.

   A: YES B: NO

   (viii) If $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are differentiable vector functions, then $\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}'(t)$.

   A: YES B: NO

   (ix) If $\mathbf{r}(t)$ is a differentiable vector function, then $\frac{d}{dt} \|\mathbf{r}(t)\| = \|\mathbf{r}'(t)\|$.

   A: YES B: NO

   (x) If $\|\mathbf{r}(t)\| = 1$ for all t, then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$ for all t.

   A: YES B: NO

   (xi) If $\mathbf{u}$ and $\mathbf{v}$ are vector functions that possess limits as $t \to a$, then

   $$\lim_{t\to a}[\mathbf{u}(t) \times \mathbf{v}(t)] = [\lim_{t\to a} \mathbf{u}(t)]\mathbf{v}(a) + \mathbf{u}(a)\,[\lim_{t\to a} \mathbf{v}(t)]\,.$$

   A: YES B: NO

   (xii) If $\mathbf{r}(t)$ is a two times differentiable vector function, then

   $$\frac{d}{dt}[\mathbf{r}(t) \times \mathbf{r}'(t)] = \mathbf{r}(t) \times \mathbf{r}''(t)\,.$$

   A: YES B: NO

2. If $\mathbf{i}, \mathbf{j}, \mathbf{k}$ denote the standard basis vectors in V_3, then the vector $2\mathbf{i} \times (3\mathbf{j} - 4\mathbf{k})$ is equal to

A: $24\mathbf{i} \cdot \mathbf{j} \cdot \mathbf{k}$	B: $-2\mathbf{k}$	C: $8\mathbf{j} + 6\mathbf{k}$	D: $6\mathbf{k} - 8\mathbf{j}$	E: $\mathbf{0}$

3. Given points $A(0, -2, 0)$, $B(1, 0, 0)$, and $C(0, 0, 1)$, the area of the triangle ABC is equal to

A: 4	B: $\frac{3\sqrt{2}}{2}$	C: 3	D: $\frac{3}{2}$	E: 0

4. If $\mathbf{u} = \langle 3, 0, 4 \rangle$, $\mathbf{v}$ lies in the xy-plane, $|\mathbf{v}| = 5$, and $\text{comp}_{\mathbf{u}}\mathbf{v} = 3$, then $\mathbf{v}$ is

A: $\langle 5, 0, 0 \rangle$	B: $\langle 9, 0, 12 \rangle$	C: $\langle \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}, 0 \rangle$	D: $\langle 3, 4, 0 \rangle$	E: there is no such vector

5. The distance between the planes $2x + y - 2z = 0$ and $4z - 2y - 4x = 6$ is

A: 1	B: 2	C: 3	D: 6	E: 0 (the planes intersect)

6. The curve $\mathbf{r}(t) = t\mathbf{i} + (2t - t^2)\mathbf{k}$ intersects the surface $z = x^2 + y^2$ at the points

A: $P_1(0, 0, 0)$, $P_2(1, 0, 1)$, and $P_3(-1, 0, 1)$	B: $P_1(1, 0, 1)$ and $P_2(-1, 0, 1)$

C: $P_1(0, 0, 0)$ and $P_2(1, 0, 1)$	D: at infinitely many points	E: at no point

7. Find an equation of the plane that contains the line $x - 1 = 2 - y = \dfrac{4 - z}{3}$ and is parallel to the plane $5x + 2y + z = 2023$.

8. Find an equation of the plane that passes through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$, and is perpendicular to the plane $x + y - 2z = 1$.

9. If $\mathbf{w} = \|\mathbf{u}\|\, \mathbf{v} + \|\mathbf{v}\|\, \mathbf{u}$, where $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are all non-zero vectors, show that $\mathbf{w}$ bisects the angle between $\mathbf{u}$ and $\mathbf{v}$.

10. Find the unit tangent vector of the curve $\mathbf{r}(t) = \langle t, \dfrac{t^2}{2}, t^2 \rangle$ at the point $t = 0$.

11. Find the arc length of the curve given by $\mathbf{r}(t) = \langle 2\sin t, 5t, 2\cos t \rangle$, $t \in [0, \pi]$.