

Problem Set 2
September 19, 2021

All numbered exercises are from the textbook *Lectures on Real Analysis*, by F. Larusson.

1. Exercise 6.6. Further, for those $a \in \mathbb{R}$ for which there exists $g : \mathbb{R} \rightarrow \mathbb{R}$ with $g' = f$, give an explicit formula for g .
2. Let $f : (0, 1) \rightarrow \mathbb{R}$ be defined by the formula

$$f(x) = \sum_{k \in \mathbb{N}, k > \frac{1}{x}} \frac{1}{2^k}.$$

- (a) Find the limits $\lim_{x \rightarrow 0^+} f(x)$ and $\lim_{x \rightarrow 1^-} f(x)$.
 - (b) Prove that f is (weakly) increasing.
 - (c) Prove that f is not a derivative; that is, there is no function $g : (0, 1) \rightarrow \mathbb{R}$ such that $g' = f$.
3. Exercise 6.8.
 4. Exercise 6.9.
 5. Exercise 6.3.
 6. Exercise 6.10.
 7. Exercise 6.12.
 8. Exercise 6.13.
 9. Exercise 6.15.
 10. Exercise 6.16(a).