Problem Set 7

November 7, 2021

All numbered exercises are from the textbook *Lectures on Real Analysis*, by F. Larusson.

1. Review Theorem 8.26.

2. Exercise 8.2.

3. Review Definition 8.28 and Theorem 8.29.

4. Exercise 8.3.

5. Exercise 8.4.

6. Exercise 8.24.

7. Review Theorem 9.5.

8. Let $X = \mathbb{R}^2$, let $O = (0,0)$ denote the origin, and let d_2 denote the Euclidean metric in X. For $P, Q \in X$, define

$$d(P,Q) = \begin{cases} d_2(P,Q), & \text{if } P, Q, \text{ and } O \text{ are colinear} \\ d_2(P,O) + d_2(O,Q), & \text{otherwise.} \end{cases}$$

Prove that d is a metric on X.

9. Let $X = \mathbb{R}^2$, and let d_2 denote the Euclidean metric in X. For $P \in X$, let $\pi(P)$ denote its projection onto the x-axis (i.e., if $P = (x_p, y_p)$, then $\pi(P) = (x_p, 0)$). For $P, Q \in X$, define

$$d(P,Q) = \begin{cases} d_2(P,Q), & \text{if } \pi(P) = \pi(Q) \\ d_2(P,\pi(P)) + d_2(\pi(P),\pi(Q)) + d_2(\pi(Q),Q), & \text{otherwise.} \end{cases}$$

Prove that d is a metric on X.