The University of Western Ontario
Department of Mathematics

Math 9024A
COMPLEX ANALYSIS I

PRESENTATION TOPICS FOR FALL 2025

1. Cauchy-Dixon Theorem (as stated in class).

Task: Proof of the theorem.

References: [A], [D].

2. Existence of analytic logaritms.

Statement: If f is analytic in a simply-connected domain D and $f(z) \neq 0$ for all $z \in D$, then f has an analytic logarithm in D; i.e., there is a function $g \in \mathcal{O}(D)$ such that $\exp(g(z)) = f(z)$ for all $z \in D$.

Task: Proof + example + discussion of what can go wrong if D not simply-connected.

References: [A], [L].

3. Corollary of Residue Theorem for calculating improper integrals.

Statement: Let f be analytic on the closed upper half-plane $\{z : \mathrm{Im}(z) \geq 0\}$ except for a finite number of points $z_1, \dots, z_k$ with $\mathrm{Im}(z_j) > 0$. Suppose there are constants $M > 0$, $R > 0$ and $\alpha > 1$ such that $|f(z)| < \frac{M}{|z|^\alpha}$ for all z with $|z| > R$. Then, the integral $\displaystyle\int_{-\infty}^{+\infty} f(x)\,dx$ is convergent and

$$\int_{-\infty}^{+\infty} f(x)\,dx = 2\pi i \cdot \sum_{j=1}^{k} \mathrm{Res}(f; z_j)\,.$$

Task: Proof + example of application.

References: [A], [L].

Suggested sources:

[A] J. Adamus, Lecture notes

[D] J. Dixon, *A brief proof of Cauchy's integral theorem*, Proc. Amer. Math. Soc. **29** (1971), 625–626.

[L] S. Lang, "Complex Analysis" 4th edition, GTM 103, Springer, 1999.