**Problem Set 1**
January 14, 2024.

**1.** Let $X$ be a non-empty finite set, and let $\mathcal{A}$ be a $\sigma$-algebra on $X$. Consider a relation on $X$:

$$x \sim y \Longleftrightarrow [x \in A \Leftrightarrow y \in A, \text{ for all } A \in \mathcal{A}] .$$

(i) Show that the above is an equivalence relation on $X$.

(ii) Show that, for every $x \in X$, its equivalence class satisfies $[x]_\sim = \bigcap\{A \in \mathcal{A} : x \in A\}$ and $[x]_\sim \in \mathcal{A}$.

(iii) Let $E_1, \ldots, E_k$ be all the distinct equivalence classes in $X$ modulo $\sim$. Show that $\mathcal{A}$ consists precisely of the empty set and unions of all sub-collections of $\{E_1, \ldots, E_k\}$ (i.e., $A \in \mathcal{A}$ iff $A = \varnothing$ or there exist $1 \leq l \leq k$ and $\{i_1, \ldots, i_l\} \subset \{1, \ldots, k\}$ such that $A = E_{i_1} \cup \cdots \cup E_{i_l}$).

**2.** Exercises 2.1 and 2.3–2.8 from the text.

**3.** Exercises 3.1, 3.2, 3.4, 3.6 and 3.8 from the text.