Problem Set 8
March 17, 2024.

1. Exercise 12.1.

2. Exercise 12.3.

3. Exercise 12.4.

4. Exercise 12.6.

5. Exercise 12.7.

6. Let $(X, \mathcal{M})$ be a measurable space.

(a) Prove that the collection of all complex measures on $(X, \mathcal{M})$ is a complex vector space (with addition and scalar multiplication defined as $(\mu + \lambda)(E) := \mu(E) + \lambda(E)$ and $(c \cdot \mu)(E) := c \cdot \mu(E)$, for $E \in \mathcal{M}$).

(b) Let $M(X)$ denote the complex vector space of all complex measures on $(X, \mathcal{M})$. Prove that the function defined as $\|\mu\| := |\mu|(X)$ is a norm on $M(X)$.