Problem Set 9
March 24, 2024.

For the exercises in Problem 1, let μ be a positive measure on a measurable space $(X, \mathcal{M})$, and let functions $f_n, f : X \to \mathbb{R}$ be $\mathcal{M}$-measurable. We say that the sequence (f_n) *converges in measure* to f, when for every $\varepsilon > 0$

$$\lim_{n\to\infty} \mu(\{x \in X : |f_n(x) - f(x)| > \varepsilon\}) = 0\,.$$

1. Exercises 10.1–10.5.

2. Suppose $\lambda, \lambda_1, \lambda_2$ are measures on a σ-algebra $\mathcal{M}$, and μ is a positive measure on $\mathcal{M}$. Prove the following statements:

(a) If λ is concentrated on a set $A \in \mathcal{M}$, then so is $|\lambda|$.

(b) If $\lambda_1 \perp \lambda_2$, then $|\lambda_1| \perp |\lambda_2|$.

(c) If $\lambda_1 \perp \mu$ and $\lambda_2 \perp \mu$, then $(\lambda_1 + \lambda_2) \perp \mu$.

(d) If $\lambda_1 \ll \mu$ and $\lambda_2 \ll \mu$, then $(\lambda_1 + \lambda_2) \ll \mu$.

(e) If $\lambda \ll \mu$, then $|\lambda| \ll \mu$.

(f) If $\lambda_1 \ll \mu$ and $\lambda_2 \perp \mu$, then $\lambda_1 \perp \lambda_2$.

(g) If $\lambda \ll \mu$ and $\lambda \perp \mu$, then $\lambda \equiv 0$.