Presentation Topics

1. Direct image of a measure [Lang, Ch. VI, Exercises 1 and 8] & inverse image of a measure.

2. Convolution of measures [Lang, Ch. VI, Exercise 16].

3. Countable products of measure spaces [Lang, Ch. VI, Exercise 22].

4. Hausdorff outer measure (of any dimension) is a metric outer measure & 0-dimensional Hausdorff measure is the counting measure [hints in Falconer, p.7].

5. Key Lemma + corollary on Hausdorff dimension [Lecture notes; hints in Falconer, p.7].

6. A set $X \subset \mathbb{R}^n$ is Hausdorff measurable iff X is contained in a G_δ-set G with $G \setminus X$ of measure 0 iff X contains an F_σ-set F with $X \setminus F$ of measure 0 [cf. Falconer, Thm. 1.6].

7. Vitali Covering Theorem [Falconer, Thm. 1.10].

8. In $\mathbb{R}^n$, the n-dimensional Hausdorff measure coincides with Lebesgue measure [Falconer, Thm. 1.12].

References:

1. S. Lang, *Real and functional analysis*, 3rd edition, Springer, GTM 142, 1993.

2. K. J. Falconer, *The geometry of fractal sets*, Cambridge University Press, 1985.