Practice Term Test 2

0. Practice problems from PS 3.

1. State the Monotone Convergence Theorem, Dominated Convergence Theorem and Fatou's Lemma.

2. State the definitions of product σ-algebra and product measure.

3. Prove or give a counterexample:
If $f_n : \mathbb{R} \to \mathbb{R}$ are Lebesgue measurable and pointwise convergent to a function f, then $\int f = \lim_{n\to\infty} \int f_n$.

4. Prove or give a counterexample:
If $f_n : \mathbb{R} \to [0,\infty)$ are Lebesgue integrable and $\lim_{n\to\infty} \int f_n = \int f$ for some function $f : \mathbb{R} \to \mathbb{R}$, then $f_n \longrightarrow f$ a.e..

5. Suppose $(X, \mathcal{M}, \mu)$ is a measure space, f and each f_n is integrable and non-negative, $f_n \longrightarrow f$ a.e., and $\int f_n \longrightarrow \int f$ as $n \to \infty$. Prove that, for each $A \in \mathcal{M}$,

$$\lim_{n\to\infty} \int_A f_n \, d\mu = \int_A f \, d\mu \,.$$

6. Exercise 11.6.

7. Exercise 11.9.

8. Exercise 11.10.

9. Exercise 11.11.

10. (a) State the definitions of a signed measure, and its positive and negative variations.
(b) State the Hahn decomposition theorem.
(c) State the Jordan decomposition theorem.

11. Exercise 12.1.

12. Exercise 12.2.

13. Exercise 12.3.

14. Exercise 12.5.

15. Exercise 12.6.