

The University of Western Ontario  
Department of Mathematics  
**Mathematics 9607B, Winter 2025**  
**COMPLEX ANALYTIC GEOMETRY**

**Lectures:** see the timetable for time/location

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**Office Hours:** by appointment

**Course Website:** <https://www.math.uwo.ca/faculty/adamus/teaching/9607B2025/>

**Material outline:** The goal of this course is to give a concise introduction to the complex analytic geometry, with a view towards modern applications. Specifically, the local geometry of real and complex analytic mappings and the properties of their images, which is one of the main interests of current research. The following is the list of main topics to be covered:

- Analytic sets and germs - definitions and basic topological properties; dimension.
- Regular and singular loci.
- Hironaka's division algorithm.
- Weierstrass Preparation, Division and Finiteness theorems.
- Rings of germs of holomorphic functions.
- Proper projections: Remmert Proper Mapping theorem, and the local description of analytic sets.
- Analyticity of the singular locus, irreducibility and irreducible components.
- Coherence theorems of Oka and Cartan.
- Coherent sheaves and complex analytic spaces.
- Remmert Rank and Open Mapping theorems.
- Analytic tensor product and fibred product of analytic mappings.
- Hironaka flatness criterion and its recent applications.
- Effective criteria for regularity of analytic mappings (time permitting).

**Problem Sets / Evaluation:** The course mark will be based on 4-5 bi-weekly homework assignments, class participation, and a final presentation. Class attendance is required.

**Prerequisites:** No particular prerequisites are required, other than a good command of undergraduate mathematics. However, having heard of the following could not hurt: a first course in commutative algebra, basic theory of smooth manifolds, basic facts from several complex variables. (All necessary facts will be recalled in class as they appear relevant.)

**Readings:** There will be no textbook for this course. Instead, a complete set of typeset lecture notes is available through the course web site. The following readings are recommended only for further study on certain topics covered:

1. S. Abhyankar, “Local analytic geometry”, Vol. XIV Academic Press, New York-London, 1964.
2. J. Adamus, *Flatness testing and torsion freeness of analytic tensor powers*, J. Algebra **289** (2005), 148–160.
3. J. Adamus, E. Bierstone and P. D. Milman, *Geometric Auslander criterion for flatness*, Amer. J. Math. **135** (2013), 125–142.
4. J. Adamus, E. Bierstone and P. D. Milman, *Geometric Auslander criterion for openness of an algebraic morphism*, Bull. Lond. Math. Soc. **45** (2013), 1060–1064.
5. J. Adamus and H. Seyedinejad, *A fast flatness testing criterion in characteristic zero*, Proc. Amer. Math. Soc. **143** (2015), 2559–2570.
6. M. Auslander, *Modules over unramified regular local rings*, Illinois J. Math. **5** (1961), 631–647.
7. E. Bierstone and P. D. Milman, “The local geometry of analytic mappings”, Dottorato di Ricerca in Matematica, ETS Editrice, Pisa, 1988.
8. H. Cartan, *Idéaux et modules de fonctions analytiques de variables complexes*, Bull. Soc. Math. France **78** (1950), 29–64.
9. E. M. Chirka, “Complex analytic sets”, **46**. Kluwer Academic Publishers Group, Dordrecht, 1989.
10. D. Eisenbud, “Commutative Algebra with a View Toward Algebraic Geometry”, Springer, New York, 1995.
11. G. Fischer, “Complex Analytic Geometry”, Lecture Notes in Mathematics, **538**. Springer, Berlin-New York, 1976.
12. A. Galligo and M. Kwieciński, *Flatness and fibred powers over smooth varieties*, J. Algebra **232** (2000), 48–63.
13. H. Grauert and R. Remmert, “Analytische Stellenalgebren”, Springer, Berlin-New York, 1971.
14. H. Hironaka, *Stratification and flatness*, in “Real and Complex Singularities”, Proc. Oslo 1976, ed. Per Holm, Stijthof and Noordhof (1977), 199–265.
15. E. Kunz, “Introduction to Commutative Algebra and Algebraic Geometry”, Birkhäuser, Boston, 1985.
16. S. Łojasiewicz, “Introduction to Complex Analytic Geometry”, Birkhäuser, Basel, 1991.
17. J.-P. Serre, *Faisceaux algébriques cohérents*, Ann. of Math. **61** (1955), 197–278.
18. H. Whitney, “Complex analytic varieties”, Addison-Wesley, Reading, Mass.-London-Don Mills, Ont., 1972.

**Additional Statements:****Religious Accommodation**

When conflicts with a religious holiday that requires an absence from the University or prohibits certain activities, students should request an accommodation for their absence in writing to the course instructor and/or the Academic Advising office of their Faculty of Registration. This notice should be made as early as possible but not later than two weeks prior to the writing or the examination (or one week prior to the writing of the test). Please visit the Diversity Calendars posted on our university's EDID website for the recognized religious holidays:

<https://www.edi.uwo.ca>.

**Accommodation Policies**

Students with disabilities are encouraged to contact Accessible Education, which provides recommendations for accommodation based on medical documentation or psychological and cognitive testing. The policy on Academic Accommodation for Students with Disabilities can be found at:

[https://www.uwo.ca/univsec/pdf/academic\\_policies/appeals/AcademicAccommodationDisabilities.pdf](https://www.uwo.ca/univsec/pdf/academic_policies/appeals/AcademicAccommodationDisabilities.pdf).

**Academic Policies**

The website for Registrar Services is <https://www.registrar.uwo.ca/>.

In accordance with policy,

[https://www.uwo.ca/univsec/pdf/policies\\_procedures/section1/mapp113.pdf](https://www.uwo.ca/univsec/pdf/policies_procedures/section1/mapp113.pdf), the centrally administered e-mail account provided to students will be considered the individual's official university e-mail address. It is the responsibility of the account holder to ensure that e-mail received from the University at their official university address is attended to in a timely manner.

**Support Services**

Please visit the Science & Basic Medical Sciences Academic Advising webpage for information on adding/dropping courses, academic considerations for absences, appeals, exam conflicts, and many other academic-related matters: <https://www.uwo.ca/sci/counselling/>.

Students who are in emotional/mental distress should refer to Mental Health@Western (<https://uwo.ca/health/>) for a complete list of options about how to obtain help.

Western is committed to reducing incidents of gender-based and sexual violence and providing compassionate support to anyone who has gone through these traumatic events. If you have experienced sexual or gender-based violence (either recently or in the past), you will find information about support services for survivors, including emergency contacts at

[https://www.uwo.ca/health/student\\_support/survivor\\_support/get-help.html](https://www.uwo.ca/health/student_support/survivor_support/get-help.html). To connect with a case manager or set up an appointment, please contact [support@uwo.ca](mailto:support@uwo.ca).

Please contact the course instructor if you require lecture or printed material in an alternate format or if any other arrangements can make this course more accessible to you. You may also wish to contact Accessible Education at [http://academicsupport.uwo.ca/accessible\\_education/index.html](http://academicsupport.uwo.ca/accessible_education/index.html) if you have any questions regarding accommodations.

Additional student-run support services are offered by the USC, <https://westernusc.ca/services/>.