LEARNING SEMINAR ON HIGHER CATEGORY THEORY

THE UNIVERSITY OF WESTERN ONTARIO

In the presentations, we will follow the list of topics below. Each topic may account for more than one talk—it is important that speakers present their topic as completely as possible, without worrying about taking too much time. Finally, a topic can be presented by more than one person (this is recommended in particular for large topics that may take 2 or more talks).

(1) **Motivation and basic concepts.** This will include motivation for higher category theory coming from different areas (e.g. homotopy theory, derived algebraic geometry, TQFTs, etc), the definition of quasicategories, as well as basic concepts such as homotopy, equivalence, and composition in a quasicategory.
  References: [AC16, Sec. 2 and 6], [Joy08, Sec. 1], and [Gro10, Sec. 1].

(2) **Introduction to quasicategories.** This topic should introduce the Joyal model structure, in which the fibrant objects are exactly quasicategories, and give some examples, specifically those obtained via the homotopy coherent nerve functor and the simplicial localization.
  References: [Joy08, Sec. 2], [Joy09, Sec. 6 and earlier], [Jar15], and [Rez16].

(3) **The Grothendieck construction and mapping spaces in quasicategories.** This is a somewhat technical topic that needs to introduce Lurie's (straightening ⊣ unstraightening)-adjunction, different notions of mapping space, and the notion of a (co)Cartesian fibration.
  References: [Lur09, Ch. 2], [DS11], and [Rez16].

(4) **The theory of quasicategories.** The speaker(s) should discuss basic categorical notions in the quasicategorical setting, including: slices, joins, (co)limits, adjoints, and so on.
  References: [Gro10, Sec. 2], [Joy08], [Joy09, Sec. 1], [Joy02], [Lur09, Ch. 4 and a bit of 5], [Rez16], and [RV15a, RV15b, RV16].

(5) **Models of $(\infty, 1)$-categories.** This should be a survey the existing models of $(\infty, 1)$-categories, including relative categories, complete Segal spaces, and simplicial categories, giving equivalences between them, and concluding with Toën's unicity theorem.
  References: [AC16, Sec. 3–4], [Joy08, Sec. 3–5], [Ber10], [Rez01], and [Toë05].

(6) **Presentable quasicategories and relation to model categories.** This topic covers the definition, basic properties, and characterizations of presentable quasicategories, as well as their relation to (combinatorial) model categories.
  References: [Gro10, Sec. 3] and [Lur09, Ch. 5].

(7) **Higher topos theory.** The goal here is to introduce the notion of an ∞-topos, and state and explain the Giraud Theorem. Time permitting, some applications of higher topos theory should be discussed.
  References: [Lur09, Ch. 6 (and optionally 7)], [Rez05], and [TV05].

Date: Fall 2016.

References

[AC16]   Omar Antolín Camarena, *A whirlwind tour of the world of $(\infty, 1)$-categories*, Mexican mathematicians abroad: recent contributions, Contemp. Math., vol. 657, Amer. Math. Soc., Providence, RI, 2016, pp. 15–61, `doi:10.1090/conm/657/13088`.

[Ber10]   Julia E. Bergner, *A survey of $(\infty, 1)$-categories*, Towards higher categories, IMA Vol. Math. Appl., vol. 152, Springer, New York, 2010, pp. 69–83, `doi:10.1007/978-1-4419-1524-5_2`.

[DS11]   Daniel Dugger and David I. Spivak, *Mapping spaces in quasi-categories*, Algebr. Geom. Topol. **11** (2011), no. 1, 263–325, `doi:10.2140/agt.2011.11.263`.

[Gro10]   Moritz Groth, *A short course on ∞-categories*, preprint, 2010, `arXiv:1007.2925`.

[Jar15]   John F. Jardine, *Path categories and quasi-categories*, preprint, 2015, `http://www-home.math.uwo.ca/~jardine/papers/preprints/pathcat5-2.pdf`.

[Joy02]   André Joyal, *Quasi-categories and Kan complexes*, J. Pure Appl. Algebra **175** (2002), no. 1-3, 207–222, Special volume celebrating the 70th birthday of Professor Max Kelly, `doi:10.1016/S0022-4049(02)00135-4`.

[Joy08]   _____, *Notes on quasicategories*, unpublished notes, 2008, `http://www.math.uchicago.edu/~may/IMA/Joyal.pdf`.

[Joy09]   _____, *The theory of quasi-categories and its applications*, Vol. II of course notes from Simplicial Methods in Higher Categories, Centra de Recerca Matemàtica, Barcelona, 2008, 2009, `http://mat.uab.cat/~kock/crm/hocat/advanced-course/Quadern45-2.pdf`.

[Lur09]   Jacob Lurie, *Higher topos theory*, Annals of Mathematics Studies, vol. 170, Princeton University Press, Princeton, NJ, 2009.

[Rez01]   Charles Rezk, *A model for the homotopy theory of homotopy theory*, Trans. Amer. Math. Soc. **353** (2001), no. 3, 973–1007 (electronic), `doi:10.1090/S0002-9947-00-02653-2`.

[Rez05]   _____, *Toposes and homotopy toposes*, unpublished notes, 2005, `http://www.math.uiuc.edu/~rezk/homotopy-topos-sketch.pdf`.

[Rez16]   _____, *Stuff about quasicategories*, lecture notes for the course Math 595 (Higher category theory and quasicategories), 2016, `http://www.math.uiuc.edu/~rezk/595-fal16/quasicats.pdf`.

[RV15a]   Emily Riehl and Dominic Verity, *The 2-category theory of quasi-categories*, Adv. Math. **280** (2015), 549–642, `doi:10.1016/j.aim.2015.04.021`.

[RV15b]   _____, *Completeness results for quasi-categories of algebras, homotopy limits, and related general constructions*, Homology Homotopy Appl. **17** (2015), no. 1, 1–33, `doi:10.4310/HHA.2015.v17.n1.a1`.

[RV16]   _____, *Homotopy coherent adjunctions and the formal theory of monads*, Adv. Math. **286** (2016), 802–888, `doi:10.1016/j.aim.2015.09.011`.

[Toë05]   Bertrand Toën, *Vers une axiomatisation de la théorie des catégories supérieures*, K-Theory **34** (2005), no. 3, 233–263, `doi:10.1007/s10977-005-4556-6`.

[TV05]   Bertrand Toën and Gabriele Vezzosi, *Homotopical algebraic geometry. I. Topos theory*, Adv. Math. **193** (2005), no. 2, 257–372, `doi:10.1016/j.aim.2004.05.004`.