## Math 9412, Spectral Graph Theory Summer 2015

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The following topics were covered:

- 1. Week of May 4, 2015: Laplacian of weighted graphs. Green's formula for graphs. Dirichlet problem. Bounds on graph spectrum. Example: eigenvalues of cycle graphs.
- 2. Spectrum of bipartite graphs. Cayley graphs of abelian groups and their spectrum via character theory. Example: spectrum of the hypercube.
- 3. Stochastic matrices. Markov chains. Ehrenfest model. Equilibrium state for Markov chains. Ergodic theorem: existence and uniqueness of equilibrium distribution for ergodic chains. Irreducible chains. Reversible chains: detailed balance and equilibrium state. Example: Markov chains of weighted graphs.
- 4. Application to Google page ranking algorithm. Monte carlo Markov chains.
- 5. Entropy. Maximum entropy principle. Statistical mechanics on finite sets. Example: Gibbs states as a maximum entropy state. Partition function and free energy.
- 6. The Ising model. Transfer matrix method for computing the partition function in 1-d Ising model. Lack of phase transition in 1-d Ising model. Spontaneous magnetization and phase transition in 2-d Ising model (via Peierls argument). Duality in the Ising model.

- 7. Shannon source coding theorem. Axioms for entropy and uniqueness of entropy.
- 8. Spanning trees. Cauchy-Binnet formula. Matrix-tree theorem. Example: number of spanning trees of hypercubes.
- 9. Isoperimetric inequalities for manifolds. Cheeger constant. Cheeger constant for graphs as measure of connectivity, complexity, or bottle-neckdness. Cheeger inequality for graphs. Examples: complete graphs, bipartite graphs, cycles.