Representation Theory of Finite Groups

Assignment 2 (Due: May 30, 2011)
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1. Show that $\frac{\mathbb{Z}[X]}{<X^n-1>}$ and $\mathbb{Z} \oplus ... \oplus \mathbb{Z}$ (n copies) are *not* isomorphic as rings.

2. a) Show that the character $\chi : S_4 \to \mathbb{C}$,

$$\chi(g) = |Fix(g)| - 1$$

is an irreducable character.
b) Compute the character table of S_4.

3. a) Show that there is a $1-1$ correspondence between partitions of n and the conjugacy classes of S_n.
b) Let $n = p_1 + p_2 + ... + p_k$, $1 \leq p_1 \leq p_2 \leq ... \leq n$, be a partition of n. Find the size of the conjugacy class defined by this partition.

4. (Exterior and symmetric powers) a) Let $\pi : G \to GL(V)$ be a representation of G. Show that $\bigwedge^2 V$ and $S^2 V$ are invariant under the action of G.
b) Show that the characters of these represenations are given by

$$\chi_{\bigwedge^2 \pi}(g) = 1/2((\chi_\pi(g))^2 - \chi_\pi(g^2)),$$
$$\chi_{S^2\pi}(g) = 1/2((\chi_\pi(g))^2 + \chi_\pi(g^2)).$$

5. Without using the sum of squares formula, show directly that any irreducible representation of a finite group is finite dimensional.

6. Let $\hat{G}$ denote the set of irreducible representations of a finite group G. Show that there is a one-to-one correspondence between $\widehat{G \times H}$ and $\hat{G} \times \hat{H}$.