WESTERN UNIVERSITY
DEPARTMENT OF MATHEMATICS
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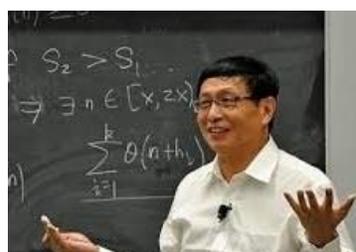
ELEMENTARY NUMBER THEORY – Mathematics 3150b

Instructor:	Ján Mináč (also known as Professor Maniac ☺)
Email:	minac@uwo.ca / and / jminac1811@gmail.com
Office Hours:	Will be discussed in class, but this will not be the main topic of the class. (My indoor office is in Middlesex College. The number of my office is a 32nd prime number. Exercise 0: Find the number of my office. (You can find the solution in the rotunda in Middlesex College.) I also use the university campus as a large, outdoor office. ☺)
Office Telephone:	(519) 661-2111, extension 86519
Class Times:	Mondays / Wednesdays / Fridays 9:30 a.m. – 10:30 a.m.
Class Location:	Middlesex College room 108
Post-Class Time Discussions immediately after class: (not required, but just for FUN! ☺)	Immediately after class, some extremely exciting, enthusiastic, informal discussions and/or consultations will follow in MC room 106 on Mondays and Wednesdays; and on Fridays immediately after class we meet in MC room 107.
Prerequistes:	Some very basic knowledge of linear algebra is useful. Also an interest in the magic of mathematics is very welcome. The courage to try and write neat proofs will be encouraged. If you are unsure about the background, please speak with me or email me.
Evaluation:	Homework 60% and Final Examination 40%
Some Questions Whose Answers we Master:	What are the prime divisors of the numbers $n^2 + 1$, $n^2 - 5$, with n a natural number? What are solutions of the equation $x^2 + y^2 = z^2$ in natural numbers? What is the largest power of 2 dividing $n!$?
Fun:	During the entire semester. ☺ And even afterwards. ☺
Number Theory Claim:	Number theory is delightful magic, filled with surprises and interesting, challenging problems. (This claim will be proved in class.)
The Art of Studying and Research:	We shall discuss and practice and enjoy the art of studying and research.

Textbook: *An Introduction to the Theory of Numbers*, by Ivan M. Niven, Herbert S. Zuckerman, Hugh L. Montgomery, Fifth Edition, John Wiley & Sons, Inc., 1991, ISBN 0-471-62546-9.

Note: I will request that the Taylor Library place their copy of this book (Fifth Edition) on reserve loan to be available for students taking this course.

This is a great book! It is available at the Western Book Store; and I hope also in the UWO Used Book Store. (This book is optional – recommended but not required, as the main source of information will be your lecture

notes.) This book contains a wealth of charming and beautiful mathematics explained in a clear and friendly way. We shall first concentrate on chapters 1, 2, 3, 4, 5, 8 and 9. We shall also review a wealth of literature, and enjoy a great number theory web site maintained by Professor Keith Matthews (http://www.numbertheory.org/keith.html). Then we shall play with algebraic numbers and ideal numbers; and view them as secret factors of ordinary numbers. We shall split "usual" primes into "complex primes". What fun! What a source of pleasant surprises!

Course Outline: The fundamental theorems of arithmetic, distribution of primes, congruences, Fermat's Little Theorem, Wilson's Theorem, the quadratic reciprocity law, primitive roots modulo powers of prime numbers, the sums of squares, Riemann zeta functions, elliptic curves, Diophantine equations, arithmetic functions, and some *ad hoc*, interesting and fascinating material enhancing these basic subjects.
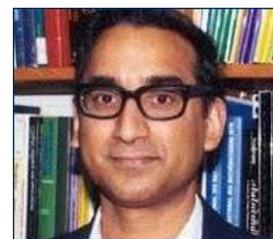

| M. Mirzakhani | L. Euler | J. Birman | J. Labute | R. Murty |

This will be the core of the course which we shall try to develop with many details and we shall aim at a full understanding of the basics.

In the other part of this course we shall bravely make an attempt to understand the "big picture" and the latest developments such as the Wiles-Taylor proof of Fermat's Last Theorem, the Langlands program, Green and Tao's theorem on primes in arithmetical sequences, Y. Zhang's theorem on infinite numbers of pairs of primes whose difference is less than 70,000,000, and later refinements, and topics in arithmetic geometry. This will often be done with bold strokes of our brushes, aiming our imagination and letting it loose; as well as freeing us from worries about details. Perhaps some of you will decide to climb this hill or that hill, or to examine certain deep topics with a microscope and the obsession that such an examination may require.

Remark. One reason why humankind may be proud today is the development of number theory. In the past only a few, usually the most outstanding mathematicians, devoted their energy to this fascinating topic. Today this theory is becoming quite popular and remarkable success was obtained in recent years. I will try to systematically develop parts of the theory and to preserve some charm, beauty, and romantic dreams of the subject.

When I was a teenager, I often daydreamed and lived in my own world full of fantasy and magic. Sometimes prime numbers snuck into this world. They were puzzling and fascinating. And then I learned that there were others dreamers called mathematicians, who explore this fantasy world in a detailed way. And what a world this is! I long to share this world with you.

Further Information:

Final Examination: There will also be one written final examination.

Final Examination Conflicts:

If you have a conflict with another final examination, you must contact the Registrar's Office as soon as possible to arrange a special time/place to write the final examination.

If you have three final examinations in 3 consecutive periods, you must contact the Dean of your faculty.

In all cases please let your instructor know.

If you miss the final examination due to illness, you must present a doctor's note to the appropriate Dean's Office, and you will be given a make-up final examination as soon as possible after the regular examination. If you are in this situation, you should contact your instructor as soon as you realize that you will miss the exam. Also see the University's policy on final examination conflicts:

https://www.uwo.ca/univsec/pdf/academic_policies/exam/conflicts.pdf

Further Information:

Academic dishonesty: Scholastic offences are taken seriously and students are directed to read the official policy: https://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_undergrad.pdf. (See also the section "Scholastic discipline for undergraduate students" of the Academic Calendar.)

Accessibility Statement: Please contact the course instructor if you require material in an alternate format or if you require any other arrangements to make this course more accessible to you. You may also wish to contact Services for Students with Disabilities (SSD) at 661-2111, extension 82147 for any specific question regarding an accommodation.

Support Services: Learning skills counsellors at the Student Development Centre (http://www.sdc.uwo.ca/) are ready to help you improve your learning skills. Students who are in emotional / mental distress should refer to Mental Health@Western for a complete list of options about how to obtain help. Additional student-run support services are offered by the University Students' Council (https://westernusc.ca/). The website for Registrarial Services is: http://www.registrar.uwo.ca.

Eligibility: You are responsible for ensuring that you have successfully completed all course prerequisites and that you have not taken an antirequisite course. Unless you have either the requisites for this course or written special permission from your Dean to enroll in it, you may be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisites.