Problem. Suppose R is the region inside the unit circle above the line x + y = 1. Calculate

$$\iint_R \frac{1}{(\log x)^2 + (\log y)^2} \, \frac{dx \, dy}{xy}.$$