Introduction to Lie groups, Math 9147B

Winter term (January-April 2020); Tu 1:30-3:30, Th 2:30-3:30; MC 108

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There will be weekly office hours. Outside of these hours it is best to send a quick e-mail before stopping by.

This course will be about finite-dimensional real Lie groups and Lie algebras. This is an incredibly rich subject that originated from work of Sophus Lie. Lie theory was extensively developed due to efforts of many prominent mathematicians, including Cartan, Chevalley and Weyl.
Lie groups are groups with additional structure: they are manifolds, and multiplication and inversion are smooth maps.
Examples:
$U(1)$, the multiplicative group of complex numbers of modulus 1, is a Lie group (one-dimensional, abelian, isomorphic to a circle);
$SU(2)$, the group of 2×2 complex unitary matrices with determinant 1, is a nonabelian compact Lie group.

Properties of a Lie group are tied to the properties of its Lie algebra. There is also an abstract concept of a Lie algebra (defined without any relation to a Lie group).

Lie groups play an important role in many areas of mathematics, including differential geometry, complex geometry, analysis, automorphic forms, number theory. Automorphism groups of geometric objects are typically Lie groups.

Approaching Lie groups at a basic level, one can study their algebraic properties (e.g. which are simple ? nilpotent ?), topological properties (given a Lie group, how many connected components does it have ? what is the fundamental group ?), or function spaces on Lie groups (e.g. L^2). These come together and serve as tools for representation theory of Lie groups and Lie algebras, which has been at the forefront of mathematics for decades.

The goal is for everyone to end the term with a working knowledge of basic concepts, including:

Lie group, the Lie algebra of a Lie group

the exponential map

the adjoint representation

Killing form

Baker-Campbell-Hausdorff formula

Lie algebra, Ado's theorem

universal enveloping algebra, Poincaré-Birkhoff-Witt theorem

semisimple and simple Lie groups

nilpotent Lie groups, solvable Lie groups

Haar measure

irreducible representations, unitary representations

Lie's theorem, Engel's theorem

Schur's lemma, Peter-Weyl Theorem

representations of $SU(2)$

If time allows, then complex Lie groups and complex Lie algebras, complexification, and real forms will also be discussed.

Marks will be based on homework assignments, one in-class test, and one presentation (every course participant will be expected to give a blackboard/chalk presentation during regular class time and hand in the presentation notes, neatly prepared in TeX).

The list of available topics will be distributed early in the term.

The UWO library has these books available *online*:

D. Bump "Lie groups" 2nd. ed.

J. Hilgert, K. Neeb "Structure and geometry of Lie groups"

C. Procesi "Lie groups. An approach through invariants and representations."

M. Sepanski "Compact Lie groups"

There are classical references including the Bourbaki books and

V. Varadarajan "Lie groups, Lie algebras, and their representations",

as well as more modern references. In particular, you may like

A. Knapp "Lie groups beyond an introduction." 2nd ed.

Relevant differential geometry references include

J. Lafontaine "An introduction to differential manifolds" *(available online at the UWO library website)*

J. Lee "Introduction to Smooth Manifolds" 2nd ed. *(available online at the UWO library website)*

F. Warner "Foundations of differentiable manifolds and Lie groups"

The prerequisites for this course are group theory and basic differential geometry. Please contact me if unsure.

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Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at this website:
`https://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_grad.pdf`

Medical Accommodation:

If you are unable to meet a course requirement due to illness or other serious circumstances, you must ensure that you have valid medical or other supporting documentation. The Student Medical Certificate is available at
`https://www.uwo.ca/univsec/pdf/academic_policies/appeals/medicalform.pdf`

It is the student's responsibility to request accommodation and make alternative arrangements with the instructor.

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See http://www.sdc.uwo.ca/ssd/