Mathematics 9616, Random Matrix Theory and Topological Recursion
January-April 2020, UWO
Instructor: Masoud Khalkhali

Course outline: Random matrix theory (RMT) concerns itself with the study of eigenvalue and eigenvector statistics of ensembles of matrices of large size. As such the subject has found many applications in science and engineering where one models the problem at hand with a random matrix of which only a minimal information is available. It has long been found that as the size of matrices grow some universal eigenvalue patterns will emerge. The subject has its roots in the work of Wishart (principal component analysis in multivariate statistics), Wigner (level statistics of neutron scattering by heavy nuclei), and Dyson. Topological recursion has its roots in the work of Tutte (combinatorics of planar maps) and Mirzakhani's recursion for the volumes of moduli spaces of curves under the Weil-Petersson metric. These ideas have been formalized and vastly generalized in the work of Eynard-Orantin and others, where it is now applied not just to random matrices, but also to a whole array of counting problems arising, for example, as matrix integrals.

Topics will include:
Matrix ensembles, Dyson 3-fold way, GUE, GOE, GSE. Wigner ensembles.
Wigner semicircle law for GUE and for Wigner ensembles, universality.
Orthogonal polynomial techniques, Harrer-Zagier formula, orbifold Euler charasteristic of moduli spaces of curves.
Matrix models, genus expansion, connections with Riemann surfaces.
Saddle point methods, resolvent techniques.
Schwinger-Dyson equations and topological recursion.

Textbooks: Main textbook: Counting Surfaces (Eynard).
Graphs on Surfaces and Their Applications (Lando and Zvonkine)
An Introduction to Random Matrices (Anderson, Guionnet, Zeitouni)

Time and place: Monday and Wednesday, 2:30-4:30 PM, MC 107

Grading: To be discussed in class.