MATH 9161A - Differential Geometry – Fall 2025 Course Outline

1. Course Information

List of Prerequisites
Multivariable Calculus (Math 4155), Introduction to Abstract Algebra (Math 3020), or equivalent.

Unless you have either the prerequisites for this course or written special permission from the Department of Mathematics to enroll in it, you may be removed and withdrawn from this course in accordance with university policy. This may be done after the add/drop deadline of the academic term, and the course will be marked as withdrawn (WDN) on your academic record. This decision may not be appealed.

Although we will quickly go over basic manifold theory, some familiarity with manifolds, differential forms, and vector fields (as covered in Math 9055A/4155A - Calculus on Manifolds) is recommended. We will emphasize the geometrical aspects of the theory and we will minimize the use of algebraic topology machinery.

2. Instructor Information

Instructor: Martin Pinsonnault
Office: redacted
E-mail: mpinson@uwo.ca

Communication Protocol: Students must use their Western **(@uwo.ca)** email addresses when contacting the instructor. To ensure timely responses and proper organization, **please include "MATH-9161" in the subject line of every email.**

3. Course Syllabus, Schedule, Delivery Mode

Official Description: Smooth manifold structure, tensors, Lie derivative, vector bundles, connections, curvature, principal bundles, distributions, Frobenius theorem, Lie groups, classical homogeneous spaces.

Syllabus: The aim of this course is to provide an introduction to differential and Riemannian geometry, with an emphasis on curvature. We will cover the foundations:

- Manifolds, tangent and cotangent bundles, tensors, Lie derivative.

- Riemannian metric and distance. Model spaces: Euclidean, hyprbolic, and spherical geometries. Homogeneous spaces.
- Levi-Civita connection and covariant derivative, geodesics, parallel transport, exponential map.
- Completeness: Hopf-Rinow theorem.
- Curvature: Riemannian, Ricci, scalar, and sectional.
- 1st and 2nd variation formulae, Jacobi fields and conjugate points.
- Gauss-Bonnet theorem.
- Comparison theorems: Myers and Cartan-Hadamar theorems.

Schedule: Tuesdays 09h30 - 10h30 and Thursdays 09h30 - 11h30 in MC 107.

Learning Outcomes: Upon successful completion of the course, students should:

- Understand essential concepts of differential geometry such as manifolds, vector bundles, and tensors.
- Master definition and basic properties of Riemannian metrics, and understand metric-induced structures (length, angle, volume).
- Practice tensor and metric computations on familiar manifolds.
- Master the definition and properties of connections on vector bundles.
- Master the definition and properties of the Riemann curvature tensor and of other curvature concepts.
- Know how to compute curvature on simple Riemannian manifolds.
- Understand how curvature affects geodesic behavior.
- Connect local and global geometry via the exponential map.
- Understand how curvature bounds affect global topological properties.
- Construct rigorous, well-structured proofs incorporating fundamental concepts from differential geometry.
- Deliver clear, coherent proof presentations to the class with appropriate mathematical exposition.
- Engage in substantive mathematical discussions with peers on course topics and related concepts.
- Navigate literature in differential geometry with confidence and comprehension.

Key Sessional Dates:
- September 4, 2025: Fall term begins.
- September 12, 2025: Last day to add / drop a first-term half course (without showing WDN).
- September 30, 2025: National Day for Truth and Reconciliation. No classes.
- October 13, 2025: Thanksgiving. No classes.
- November 3 -- 9, 2025: Fall Reading Week.
- December 1, 2025: Last day to withdraw from a half course without academic penalty.
- December 9, 2025: Classes End.
- December 10, 2025: Study Day.
- December 11 -- 22, 2025: Exam period.

4. Course Materials

We will closely follow the following **required** textbook:

- John M. Lee, *Introduction to Riemannian Manifolds*, Graduate Texts in Mathematics **176**, 2nd ed., Springer, 2018.

We will also draw from other well-known references as listed below. In order to develop a deeper understanding of the subject matters, we encourage students to use more than one reference. When using online references, students must keep in mind that they may contain typos and other inaccuracies.

- Barrett O'Neill, *Semi-Riemannian geometry: With applications to relativity*, Pure and Applied Mathematics, vol. **103**, Academic Press, 1983.

  This comprehensive text requires no prior knowledge of differential geometry, opening with a thorough introduction to smooth manifolds and tensor fields. It covers all the material that will be presented in the course.

- B. Chow and Y. Chow, *Lectures on Differential Geometry*, Graduate Studies In Mathematics **245**, AMS, 2024.

  This text begins with an extensive treatment of the Riemannian geometry of submanifolds of $\mathbf{R}^n$, then progresses through the foundations of Riemannian geometry. The final section provides a comprehensive introduction to geometric analysis, culminating in a discussion of geometric flows.

- Frank Warner, *Foundations of Differential Manifolds and Lie Groups*, Graduate Texts in Mathematics **94**, 2nd ed., Springer, 1983.

  This book does not cover Riemannian geometry but gives a neat and concise introduction to manifolds and Lie groups.

Costs of Textbooks on the Course Syllabus

The required textbook (Lee) can be ordered from Springer for approximately 65$ CAD.

Website: All other course material will be posted to OWL: https://westernu.brightspace.com/

Students are responsible for checking the course OWL site (https://westernu.brightspace.com/) regularly for news and updates. This is the primary method by which information will be disseminated to all students in the class.

If students need assistance with the course OWL site, they can seek support on the OWL Brightspace Help page. Alternatively, they can contact the Western Technology Services Helpdesk. They can be contacted by phone at 519-661-3800 or ext. 83800.

5. Methods of Evaluation

Grading Scheme and Assessment Dates

The overall course grade will be calculated as listed below:
- 6 short homeworks (every other week): 30%
- Final exam: 40%
- Presentation: 15%
- Written report: 15%.

The homeworks are an integral part of the course and special attention must be paid to the presentation. They will be evaluated based on both correctness and clarity. Students are allowed to discuss assignment problems, but each student should write their solutions individually, with proper credit given to those who helped you and to any references used.

At the end of the semester, each student will give a presentation (50 minutes) on a subject related to the course material. The topic should be selected as early as possible. We will maintain a list of possible topics that students can choose from. Alternatively, students may decide on another topic of particular interest/relevance for them after discussion with the instructor. The presentations cannot use slides, must include significant results, and proofs of some of them. Students will have to write a report prior to their presentation and to hand it in (on paper or by email) **at least two weeks before the talk** in order to get preliminary comments and to make corrections if necessary. The final version should be given or emailed to the instructor before the end of the exam week.

Use of Generative AI Tools

Generative AI tools (e.g., ChatGPT, Claude.ai, Copilot, Gemini) **are not permitted.**

General information about missed coursework
Students must familiarize themselves with the *University Policy on Academic Consideration – Undergraduate Students in First Entry Programs,* posted on the Academic Calendar: https://www.uwo.ca/univsec/pdf/academic_policies/appeals/academic_consideration_Sep24.pdf,

This policy does not apply to requests for Academic Consideration submitted for **attempted or completed work**, whether online or in person.

The policy also does not apply to students experiencing longer-term impacts on their academic responsibilities. These students should consult Accessible Education.

For procedures on how to submit Academic Consideration requests, please see the information posted on the Office of the Registrar's webpage:
https://registrar.uwo.ca/academics/academic_considerations/
All requests for Academic Consideration must be made within 48 hours after the assessment date or submission deadline.

All Academic Consideration requests must include supporting documentation; however, recognizing that formal documentation may not be available in some extenuating circumstances, the policy allows students to make one Academic Consideration request **without supporting documentation** in this course. However, the following assessments are excluded from this, and therefore always require formal supporting documentation:

- Final Examination
- Presentation

When a student *mistakenly* submits their one allowed Academic Consideration request **without supporting documentation** for the assessments listed above or those in the **Coursework with Assessment Flexibility** section below, the request cannot be recalled and reapplied. This privilege is forfeited.

Evaluation Scheme for Missed Assessments
A penalty of 10% per day will be apply for late homeworks, for a maximum of 3 days, after which a mark of 0 will be given (unless proper Academic Consideration is granted). If a student misses an assessment with appropriate documentation, the weight of the missed assessment will be redistributed to the final examination.

When a student misses the Final Exam and their Academic Consideration has been granted, they will be allowed to write the Special Examination (the name given by the University to a makeup Final Exam). See the Academic Calendar for details (under Special Examinations), especially for those who miss multiple final exams within one examination period.

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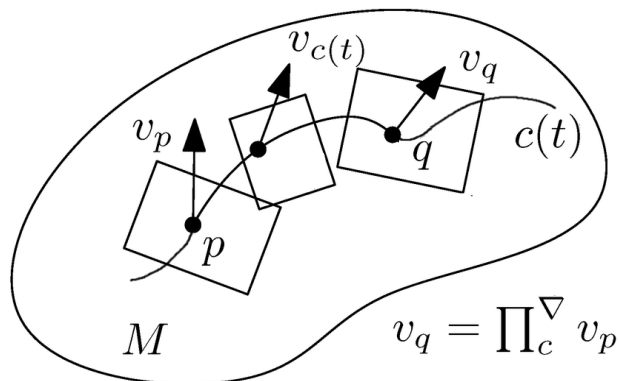
6. Additional Statements

6.1 Religious Accommodation
When conflicts arise with a religious holiday that requires an absence from the University or prohibits certain activities, students should request an accommodation for their absence in writing to the course instructor and/or the Academic Advising office of their Faculty of Registration. This notice should be made as early as possible, but not later than two weeks prior to the writing of the examination (or one week prior to the writing of the test).

Please visit the Diversity Calendars posted on our university's EDID website for the recognized religious holidays - https://www.edi.uwo.ca

6.2 Academic Accommodation Policies

Students with disabilities are encouraged to contact Accessible Education, which provides recommendations for accommodation based on medical documentation or psychological and cognitive testing. The policy on Academic Accommodation for Students with Disabilities can be found at:

https://www.uwo.ca/univsec/pdf/academic_policies/appeals/Academic Accommodation_disabilities.pdf.

6.3 General Academic Policies
The website for Registrar Services is https://www.registrar.uwo.ca/.

Use of @uwo.ca email: In accordance with policy, https://www.uwo.ca/univsec/pdf/policies_procedures/section1/mapp113.pdf, the centrally administered e-mail account provided to students will be considered the individual's official university email address. It is the responsibility of the account holder to ensure that emails received from the University at their official university address are attended to in a timely manner.

Requests for Relief (formally known as "appeals")

Policy on Request for Relief from Academic Decision: https://uwo.ca/univsec//pdf/academic_policies/appeals/requests_for_relief_from_academic_decisions.pdf

Procedures on Request for Relief from Academic Decision (Undergraduate): https://uwo.ca/univsec//pdf/academic_policies/appeals/undergrad_requests_for_relief_procedure.pdf

Procedures on Request for Relief from Academic Decision (Graduate): https://uwo.ca/univsec//pdf/academic_policies/appeals/graduate_requests_for_relief_procedure.pdf

6.4 Scholastic Offences

Policy on Scholastic Offences: https://uwo.ca/univsec//pdf/academic_policies/appeals/scholastic_offences.pdf

Procedures on Scholastic Offences: https://uwo.ca/univsec//pdf/academic_policies/appeals/undergrad_scholastic_offence_procedure.pdf

Use of Electronic Devices During Assessments

In courses offered by the Faculty of Science, the possession of unauthorized electronic devices during any in-person assessment (such as tests, midterms, and final examinations) is strictly prohibited. This includes, but is not limited to: mobile phones, smart watches, smart glasses, and wireless earbuds or headphones.

Unless explicitly stated otherwise in advance by the instructor, the presence of any such device at your desk, on your person, or within reach during an assessment will be treated as a *scholastic offence*, even if the device is not in use.

Only devices expressly permitted by the instructor (e.g., non-programmable calculators) may be brought into the assessment room. It is your responsibility to review and comply with these expectations.

Use of Generative AI Tools

Unless otherwise stated, the use of generative AI tools (e.g., ChatGPT, Microsoft Copilot, Google Gemini, or similar platforms) is **not permitted** in the completion of any course assessments, including but not limited to: assignments, lab reports, presentations, tests, and final examinations.

Using such tools for content generation, code writing, problem solving, translation, or summarization—when not explicitly allowed—will be treated as a **scholastic offence**.

If the use of generative AI is permitted for a particular assessment, the conditions of use will be specified by the instructor in advance. If no such permission is granted, students must assume that use is prohibited. It is your responsibility to seek clarification before using any AI tools in academic work.

6.5 Support Services
Please visit the Science & Basic Medical Sciences Academic Advising webpage for information on adding/dropping courses, academic considerations for absences, requests for relief, exam conflicts, and many other academic-related matters: https://www.uwo.ca/sci/counselling/.

Students who are in emotional/mental distress should refer to Mental Health@Western (https://uwo.ca/health/) for a complete list of options about how to obtain help.

Western is committed to reducing incidents of gender-based and sexual violence and providing compassionate support to anyone who has gone through these traumatic events. If you have experienced sexual or gender-based violence (either recently or in the past), you will find information about support services for survivors, including emergency contacts at

https://www.uwo.ca/health/student_support/survivor_support/get-help.html.

To connect with a case manager or set up an appointment, please contact support@uwo.ca.

Please contact the course instructor if you require lecture or printed material in an alternate format or if any other arrangements can make this course more accessible to you. If you have any questions regarding accommodations, you may also wish to contact Accessible Education at

http://academicsupport.uwo.ca/accessible_education/index.html

Learning-skills counsellors at Learning Development and Success (https://learning.uwo.ca) are ready to help you improve your learning skills. They offer presentations on strategies for improving time management, multiple-choice exam preparation/writing, textbook reading, and more. Individual support is offered throughout the Fall/Winter terms in the drop-in Learning Help Centre, and year-round through individual counselling.

Additional student-run support services are offered by the USC, https://westernusc.ca/services/.

Last updated 2025-08-29