THE UNIVERSITY OF WESTERN ONTARIO
London Ontario

Applied Mathematics Ph.D. Comprehensive Examination

25 May 2015
Part I: 9:30 am - 11:30 am

Instructions: The exam consists of Part I and Part II. Part I consists of mandatory problems and covers basic material. In Part I, 80% is required for a passing grade.

You may use a calculator, pen, and pencil. NO other aids are allowed. Your calculator must NOT be capable of wireless communication or capable of storing and displaying large text files.

PART I: Do ALL of the questions in the following four sections.

1. **Linear Algebra**

   (a) Let $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 2 & -1 \\ 2 & 4 & 6 \end{pmatrix}$.

   Find a basis for and the dimension of the range of A.

   (b) Let $q(x,y) = 5x^2 + 5y^2 + 4xy$.

   (i) Rewrite this quadratic function in matrix form.

   (ii) Find $\min q(x,y)$ and $\max q(x,y)$ on the unit circle, that is, subject to $x^2 + y^2 = 1$.

   (c) Let $B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$. Find B^{-1} or prove that it does not exist.

2. **Calculus**

   (a) Find

   $$\lim_{h \to 0} \frac{\sin^7(\frac{\pi}{6} + \frac{h}{2}) - (\frac{1}{2})^7}{h}.$$

   (b) Find the mean value $M(f)$ of $f(x) = x^3$ on the interval $2 \le x \le 4$.

   (c) Find $\int (\ln x)^2 dx$.

   (d) Find an equation for the tangent plane to the surface S with equation

   $$x^2 y + y^2 z + 3z^2 x = 53,$$

   at the point $(2, -1, 3)$.

3. **Ordinary differential equations**

   (a) Find the general solution to $ty' - y = t^2 e^{-t}$ with $t > 0$.

   (b) Find the general solution to $\vec{x}' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \vec{x}$.

   (c) Find the general solution to $y'' - y' - 2y = -2t + 4t^2$.

4. **Numerical Methods** Explicitly show how you obtain your numerical answers in the following.

   (a) Write out the Taylor series for $\cos x$ to order n about an *arbitrary* point x_0, including the form of the remainder. Then derive a formula to determine a number of terms of the series that will guarantee an error less than some given tolerance, tol.

   (b) Find a solution to $e^x - 3x^2 = 0$ numerically to 6 digit accuracy. Explain why you believe you have 6 digit accuracy (your answer should involve both backward and forward error descriptions).

   (c) Evaluate to within 10^{-2} the integral $\int_{-\sigma}^{\sigma} \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{x^2}{2\sigma^2})$. (Part marks will be given for coming up with some way of evaluating the integral, even if you don't know how accurate the result is.)

   (d) Consider the IVP $y' = ty$, on $0 \leq t \leq 1$, with $y(0) = 1$. Apply any *implicit* numerical method with a step of $h = 1/3$ to solve this problem.