Math Club At Western (MaCAW) 2nd Annual Math Contest
17 January 2019

- This contest consists of 4 questions worth 10 marks each, for a total of 40 marks. The questions in this contest are arranged in order of increasing difficulty.
- The time limit for this contest is **1 hour**.
- A **full solution** is required for each question. Marks are awarded for completeness and clarity.
- When applicable, express all answers as **simplified exact numbers**; for example, use $\pi + 2$ instead of 5.1415... , and use $1 - \sqrt{2}$ instead of –0.4142... .
- Complete **all** solutions on the lined paper provided. Write your name on the top of **every** page being submitted for grading. The lined paper provided may also be used for rough work. Do **not** include solutions and rough work on the same page.
- **No** electronic devices or paper references are permitted during the contest.
- If you have any questions, or finish the contest early, please raise your hand.
- Good luck!

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1. Determine the rightmost non-decimal digit of the number $\sqrt{2019^{1000}}$ (i.e., the ones digit).

2. A certain book contains 50 (two-sided) sheets of paper, and hence has 100 pages. These pages are labelled 1, 2, 3, . . . , 99, 100 in that order. After tearing n of the 50 sheets from the book, the sum of the page numbers showing on the remaining sheets is 4946. Determine the value of n, and show that this value is unique.

3. In triangle ΔABC, point M lies on AC and point N lies on BC, and AN and BM intersect at a unique point O, which lies in the triangle ΔABC. If $Area(\Delta AOM) = 1$, $Area(\Delta BON) = 2$, and $Area(\Delta AOB) = 3$, determine the area of triangle ΔMCN.

4. Consider the set $S = \{1, 3, 5, \ldots, 97, 99\}$, that is, S is the set of odd numbers between 1 and 99, inclusive. Let A_1 be the sum of all elements of S, let A_2 be the sum of all products of 2 distinct elements of S (ignoring the order of the elements in the product), and so on, so A_{49} is the sum of all products of 49 distinct elements of S (ignoring order), and A_{50} is the product of all elements of S. (For example, if we replaced S with $\{a, b, c, d\}$, we would have $A_2 = ab + ac + ad + bc + bd + cd$ and $A_3 = abc + abd + acd + bcd$.) Determine the value of $A_1 - A_2 + A_3 - \cdots + A_{49} - A_{50}$.