DRP Final Presentations

November 30, 2019 MC 107

Time	Speaker	Title
10:00 AM	Timothy Yau	Tangent Vectors as Derivations
10:20 AM	Yeonjoon Choi	Wick's Theorem and Gaussian Integrals
10:40 AM	Szymon Adamus	The Adventures of Matroids and the Greedy Algorithm
11:00 AM	Mahima Siali	Analysis of Diffie-Hellman Key Exchange
11:20 AM	Boyuan Pang	The Index Theorem from a Geometrical Viewpoint
11:40 AM	Charles Hau	Kirchoff's Matrix-Tree Theorem
12:00 PM		Lunch Break
01:00 PM	Amar Venga	Sheaves and their applications
01:20 PM	Jonathan Chang	Introduction to Category Theory
01:40 PM	Paul Norton	Products in Categories
02:00 PM	Jacob Fabe	Representable functors in the category of G-Sets

Abstracts

Tangent Vectors as Derivations
Timothy Yau, 10:00 AM

To generalize the concept of a tangent vector so that it can be applied to manifolds, I will show that an isomorphism exists between the set of tangent vectors at some point and the derivations at that point. To do so, I will define what a germ of functions is, show that it is an equivalence relation between functions, and define what a derivation at a point p is in relation to germs and work through examples. With these definitions, I can show that the isomorphism exists defined as a directional derivative operator.

Wick's Theorem and Gaussian Integrals
Yeonjoon Choi, 10:20 AM

In quantum mechanics and random matrix theory, computation of Gaussian integrals is extremely important. In this talk, I will go over how Wick's Theorem and graphs of pairing is used to compute simple Gaussian integrals in single and multi-variable setting. For more complicated integrals, Feynman's theorem allows us to compute asymptotic expansion by summing over automorphisms of graphs.

The Adventures of Matroids and the Greedy Algorithm
Szymon Adamus, 10:40 AM

The presentation will give a brief overview of matroids and their properties, and introduce the greedy algorithm. A proof will be given that matroids can be redefined in terms of the greedy algorithm.

Analysis of Diffie-Hellman Key Exchange
Mahima Siali, 11:00 AM

Current online security relies on the study of cryptography, which involves the creation of algorithms for encrypting and decrypting messages and data through the use of secret keys. One of the first created methods of exchanging secret keys over a public platform is called Diffie-Hellman key exchange. This

presentation aims to discuss the methods of Diffie-Hellman exchange through a live example. Additionally, the insecurities within Diffie-Hellman will be addressed, focusing on its susceptibility to Person-in-the-Middle attacks. Time permitting, the RSA cryptosystem will be introduced as a method of overcoming the flaws of Diffie-Hellman through using one way functions.

The Index Theorem from a Geometrical Viewpoint
Boyuan Pang, 11:20 AM

In this talk, I will explain the Index Theorem on the surface M and its application on the sphere S^2. The latter is also known as the Hairy Ball Theorem. This theorem connects every orientable compact surface M with its Euler's characteristic $\chi(M)$ by constructing vector field V on M. The connection is based on the sum of indices of the critical points of V, which is equal to $\chi(M)$. Applying this theorem to sphere S^2 proves the Hairy Ball Theorem, that there is no smooth vector field on S^2 which is everywhere nonzero. I will also cover some basic surfaces in R^3 and R^4 such as Torus, Klein's bottle, and Projective plane.

Kirchoff's Matrix-Tree Theorem
Charles Hau, 11:40 AM

In this presentation, we will introduce a celebrated theorem in algebraic graph theory which solves a counting problem on graphs. We will also talk about some consequences.

Sheaves and their applications
Amar Venga, 01:00 PM

This talk is intended to give a gentle introduction to sheaves, and will introduce many of the basic notions, such as sheaves and presheaves, germs, stalks, and morphisms. The talk will discuss the importance of stalk-local properties of sheaves, and will also discuss applications of sheaves to other areas of math. Some background knowledge of topology and/or category theory may augment the audience's experience, but is in no way required.

Introduction to Category Theory
Jonathan Chang, 01:20 PM

Categories arise in almost every area of pure mathematics. We will look at the definition of them and study some examples. In particular, we will show how groups actions arise from a certain class of categories.

Products in Categories
Paul Norton, 01:40 PM

In this talk, we will cover the axioms of introduce what a category is, then cover topics such as defining functors, which are certain relations between categories. We define products in this general context, and look at some examples in familiar categories.

Representable functors in the category of G-Sets
Jacob Fabe, 02:00 PM

We use the Yoneda lemma to study the action of a group on itself given by left multiplication. As an application, we recover the fact that any group embeds in the group of permutations of its underlying set.