

**The University of Western Ontario  
Department of Applied Mathematics**

**Course Outline Winter 2018**  
**Applied Mathematics 4617b/AM9566: Numerical Solution of PDE's**

**Description:** Finite difference methods, stability analysis for time-dependent problems.

**Prerequisite(s):** permission of the instructor + an ability to program in a compiled language.

**Book:** "Finite Difference Schemes and Partial Differential Equations", John C. Strikwerda, Second edition, SIAM, 2004 (required)

**Instructor:** Dr. A. MacIsaac (allanb@uwo.ca), Middlesex College, Room 285

**Lectures:** Monday, Wednesday, Friday 1:30pm - 2:20pm MC204 ( Hooray )

**Office Hours:** TBD

**Note this course has joint lectures for AM4617 and AM9566, however the Graduate student evaluation will include unique assignments and exams and may require additional reading on topics not covered in class.**

**Marking Scheme:**

40% Assignments
25% Midterm Exam
35% Final Exam

We will try to cover the following chapters at various depths.

Chapter 1: Hyperbolic Partial Differential Equations.

1. Overview of Hyperbolic Partial Differential Equations.
2. Boundary Conditions.
3. Introduction to Finite Difference Schemes.
4. Convergence and Consistency.
5. Stability.
6. The Courant-Friedrichs-Lewy Condition.

Chapter 2: Analysis of Finite Difference Schemes.

1. Fourier Analysis.
2. Von Neumann Analysis.
3. Comments on Instability and Stability

Chapter 3: Order of Accuracy of Finite Difference Schemes.

1. Order of Accuracy.
2. Stability of Lax-Wendroff and Crank-Nicolson Schemes.
3. Difference Notation and Difference Calculus.
4. Boundary Conditions for Finite Difference Schemes.
5. Solving Tridiagonal Systems.

Chapter 4: Stability for Multistep Schemes.

1. Stability for the Leapfrog Scheme.
2. Stability for General Multistep Schemes.

Chapter 5: Dissipation and Dispersion.

1. Dissipation.
2. Dispersion.
3. Group Velocity and the Propagation of Wave Packets.

Chapter 6: Parabolic Partial Differential Equations.

1. Overview of Parabolic Differential Equations.
2. Parabolic Systems and Boundary Conditions.
3. Finite Difference Schemes for Parabolic Equations.
4. The Convection-Diffusion equation.
5. Variable Coefficients

Chapter 7: Systems of Partial Differential Equations.

1. Stability of Finite Difference Schemes for Systems.
2. Finite Difference Schemes in Two and Three Dimensions.
3. The Alternating Direction Implicit Method.

Chapter 10: Convergence Estimates for Initial Value Problems.

1. Convergence Estimates for Smooth Initial Functions.
2. Related Topics.
3. Convergence Estimates for Non-smooth Initial Functions.
4. Convergence Estimates for Parabolic Differential Equations.
5. The Lax-Richtmyer Equivalence Theorem.
6. Analysis of Multistep Schemes.
7. Convergence Estimates for Second-Order Differential Equations.

Chapter 12: Elliptic Partial Differential Equations and Difference Schemes.

1. Overview of Elliptic Partial Differential Equations.
2. Regularity Estimates for Elliptic Equations.
3. Maximum Principles.
4. Boundary Conditions for Elliptic Equations.
5. Finite Difference Schemes for Poisson's Equations.
6. Coordinate Changes and Finite Differences.

Chapter 13: Linear Iterative Methods.

1. Solving Finite Difference Schemes for Laplace's Equation in a Rectangle.
2. Eigenvalues of the Discrete Laplacian
3. Analysis of the Jacobi and Gauss-Seidel Methods.
4. Convergence Analysis of Point SOR.
5. Consistently Ordered Matrices.
6. Linear Iterative Methods for Symmetric, Positive Definite Matrices.
7. The Neumann Boundary Value Problem.

Chapter 14: The Method of Steepest Descent and the Conjugate Gradient Method.

1. The Method of Steepest Descent.
2. The Conjugate Gradient Method.
3. Implementing the Conjugate Gradient Method.
4. A convergence Estimate for the Conjugate Gradient Method.
5. The Preconditioned Conjugate Gradient Method.

Addendum to all Applied Mathematics Course Outlines:

If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to your faculty's Dean's Office as soon as possible and contact your instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from your faculty's Dean's Office immediately.

For further information please see: <http://studentservices.uwo.ca/secure/index.cfm>

A student requiring academic accommodation due to illness, should use the Student Medical Certificate when visiting an off-campus medical facility or request a Records Release Form (located in the Dean's Office) for visits to Student Health Services. The form can be found here:

[https://studentservices.uwo.ca/secure/medical\\_document.pdf](https://studentservices.uwo.ca/secure/medical_document.pdf)

#### Mental Health Statement:

Students who are in emotional/mental distress should refer to Mental Health@Western <http://www.uwo.ca/uwocom/mentalhealth/> for a complete list of options about how to obtain help.

#### Accessibility:

Please contact the course instructor if you require lecture or printed material in an alternate format or if any other arrangements can make this course more accessible to you. You may also wish to contact Services for Students with Disabilities (SSD) at 661-2111 ext. 82147 if you have questions regarding accommodation.

#### Support Services:

Learning-skills counsellors at the Student Development Centre (<http://www.sdc.uwo.ca>) are ready to help you improve your learning skills. They offer presentations on strategies for improving time management, multiple-choice exam preparation/writing, textbook reading, and more. Individual support is offered throughout the Fall/Winter terms in the drop-in Learning Help Centre, and year-round through individual counselling.

Additional student-run support services are offered by the USC, <http://westernusc.ca/services>.

The website for Registrarial Services is <http://www.registrar.uwo.ca>.