

# Algebraic Number Theory

## Math 4151/9051B

### Spring 2018

**Class times and location:** Tu 9:30--11:30am and Th 9:30--10:30am in MC 107

**Prerequisites:** [Mathematics 4120A/B](#); [Mathematics 3151A/B](#) strongly recommended but not required.

## Overview

This is an introduction to algebraic number theory. Specific topics we will study include number fields and rings, algebraic numbers and integers, prime factorization in integer rings, and class and unit groups of integer rings. Depending on the available time, other potential topics are zeta functions of number fields, counting primes in integer rings, and (very basic) class field theory.

One of the leading contributors to the field was Richard Dedekind, and like many others, he was inspired by Fermat's Last Theorem. The history of the theorem is long and convoluted, and complicated by the fact that it was not proven until 1994, long after his death. As is well known (in number theory), Fermat claimed to have a proof but lacked the space to prove it, and all attempts since then to rediscover his proof have failed. We now believe that his proof assumed certain commutative rings were all principal ideal domains. However, not all relevant rings are PIDs, so the proof would have been flawed. Rather, one needs Dedekind's notion of ideal numbers to recover a suitable notion of prime factorization.

While the story of Fermat's Last Theorem will play an important role in our class, it will mostly be a secondary role as a source of inspiration.

## Instruction:

**Instructor:** [Chris Hall](#)

**Office Hours:** after class

**Text:** There are two texts for this course:

D. Cox, [Primes of the Form  \$x^2+ny^2\$](#) ;

D.A. Marcus, [Number Fields](#).

The main text is the second. I will offer suggested readings out of the first book throughout the semester.

Other nice references include:

D. Lorenzini, [An Invitation to Arithmetic Geometry](#);

G.J. Janusz, [Algebraic Number Fields](#);

S. Lang, [Algebraic Number Theory](#);

J. Neukirch, [Algebraic Number Theory](#).

These are of varying levels of difficulty, so browse a few before choosing one. (None is required though!)

## Expectations:

**Dropbox:** I will share electronic articles and notes using [Dropbox](#). Please [provide me](#) with the e-mail address you use to [login](#).

**Attendance:** Our class is small and someone's absence can greatly impact the rest of the class. Therefore you are expected to attend class or to let me know in advance when you are unable to attend.

**Exercises:** There will be eight sets of exercises assigned throughout the semester. I will post the list of exercises on OWL with a due date (typically one week after the assignment opens). You should typeset your answers and submit an electronic version via dropbox.

**Projects:** Each person must schedule a 60-minute meeting with me during the exam period Apr 14-30 for a private oral exam. Please complete [a survey](#) to indicate what times would work for you; I will be the only one who sees your answers, and I will use it during the last week of class to schedule the exams. If you need me to commit before Apr 11 to the date of your exam, please let me know asap.

To prepare for this exam you must:

1. select a topic from my list or negotiate your own topic, and obtain my approval by Mar 1;
2. read material I suggest to you;
3. prepare an outline for a 5-page document relevant to your topic, and obtain my approval by Mar 15;
4. use your outline to complete your document;
5. bring a printed copy of the latest version to your final exam.

You should certainly meet with me if you have questions or concerns about how to prepare. At the exam in April we will discuss your project and analyze your document.

## **Evaluation**

**Basis:** Your performance will be measured using two tangible items: exercises and a project. They will contribute 60% and 30% to respectively your final grade. The remaining 10% consists of a grade for the oral exam.

**Scholastic offences:** Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site:

[http://www.uwo.ca/univsec/handbook/appeals/scholastic\\_discipline\\_grad.pdf](http://www.uwo.ca/univsec/handbook/appeals/scholastic_discipline_grad.pdf)