Functional Analysis, Math 9054A/4154A

[course outline version August 21, 2020; may be updated later]

Fall term (September-December 2020); Tue 12:30-2:30, Thurs. 12:30-1:30; meeting online

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The course will be primarily about Hilbert spaces and bounded linear operators on Hilbert spaces. We will talk about Banach spaces as well.

To give an easy example, consider a <u>finite-dimensional</u> Hilbert space. It is a finite-dimensional complex vector space with an inner product. An example: $V = \mathbb{C}^n$ ($n \in \mathbb{N}$) with the standard inner product

$$\langle z, w \rangle = z_1 \bar{w}_1 + ... + z_n \bar{w}_n,$$

and a bounded linear operator from V to V is a $\mathbb{C}$-linear map $V \to V$.

Basics of Hilbert spaces is a standard part of a mathematician's education, used throughout mathematics, perhaps most often in analysis, geometry, and representation theory. These mathematical techniques are also heavily used in other disciplines, most notably in theoretical physics (especially quantum mechanics).

The official course description reads:
Hilbert spaces: L^2 spaces, orthogonal complements, dual spaces, Riesz representation theorem, the Fredholm alternative, spectral resolution of compact normal operators. Banach spaces: Hahn-Banach theorem, bounded linear operators, adjoints, closed graph and Banach Steinhaus theorems.

Marks will be based on homework assignments (60%), one midterm test given on zoom (31%), and the "level of engagement" component (9%). Graduate students (those enrolled in 9054A) will be given a somewhat harder version of the online test than the undergraduate students (those registered in 4154A). The test will be scheduled centrally, I will announce the date and time as soon as it is known.

Course textbooks are
    B. MacCluer "Elementary functional analysis"
    B. Rynne and M. Youngson "Linear functional analysis".

Both are available for free (electronically) from the UWO library website.

Here is a list of some standard functional analysis references:

J. Conway "A course in functional analysis"

V. Hutson, J. Pym "Applications of functional analysis and operator theory"

A. Kolmogorov, S. Fomin "Elements of the theory of functions and functional analysis"

M. Reed, B. Simon "Methods of mathematical physics" (vol. 1 and 2 are most relevant to this course)

W. Rudin "Functional analysis"

N. Young "An introduction to Hilbert space"

Office hours will be on zoom by appointment (send me an email and ask for a zoom meeting).

The **prerequisites** for the course are:

intermediate linear algebra (UWO Math 2120A/B)

real analysis II (UWO Math 3122A/B)

pre- or co-requisite: complex analysis (UWO Math 3124A/B)

Math 4154A students: Unless you have either the requisites for this course or written special permission from your Dean to enroll in it, you may be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisites.

Accommodated evaluations: late homework will not be accepted. Please submit any time before the deadline. If there is a proper documentation for a missed assignment, then the homework component of the grade will be adjusted accordingly. In order to obtain a passing numerical course grade, a student must write a proctored assessment. If a student missed the midterm online test and there is proper supporting documentation, then a make up test will be given.

In accordance with policy, http://www.uwo.ca/its/identity/activatenonstudent.html, the centrally administered e-mail account provided to students will be considered the individual's official university e-mail address. It is the responsibility of the account holder to ensure that e-mail received from the University at his/her official university address is attended to in a timely manner.

Technical requirements for the course: computer with a microphone and webcam, stable internet connection, a scanner or an alternative.

The remote learning sessions for this course will not be recorded. All supporting learning materials will be available on the course owl site. Attendance in remote learning sessions will be used towards the "level of engagement" course evaluation component. Some of the class meetings will rely on asynchronous course materials which students are expected to review in timely fashion.

Participants in this course are not permitted to record the sessions, except where recording is an approved accommodation, or the participant has the prior written permission of the instructor.

Tests and examinations in this course will be conducted using Zoom. You will be required to keep your camera on for the entire session, hold up your student card for identification purposes, and share your screen with the invigilator if asked to do so at any time during the exam. The exam session will not be recorded.

More information about the use of Zoom for exam invigilation is available in the Online Proctoring Guidelines at the following link:

   https://www.uwo.ca/univsec/pdf/onlineproctorguidelines.pdf

Completion of this course will require you to have a reliable internet connection and a device that meets the system requirements for Zoom. Information about the system requirements are available at the following link: https://support.zoom.us/hc/en-us
Please note that Zoom servers are located outside Canada. You will be expected to attend any Zoom sessions under your full name. If you would prefer to use only your first name or a nickname on Zoom, please discuss this with the instructor in advance of the test or examination, or class meeting.

Accommodation policies: Students with disabilities work with Accessible Education (formerly SSD) which provides recommendations for accommodation based on medical documentation or psychological and cognitive testing. The Academic Accommodation for Students with Disabilities policy can be found at:
`https://www.uwo.ca/univsec/pdf/academic_policies/appeals/Academic Accommodation_disabilities.pdf`

Western is committed to achieving barrier-free accessibility for all its members, including graduate students. As part of this commitment, Western provides a variety of services devoted to promoting, advocating, and accommodating persons with disabilities in their respective graduate program.

Graduate students with disabilities (for example, chronic illnesses, mental health conditions, mobility impairments) are encouraged to register with Student Accessibility Services, a confidential service designed to support graduate and undergraduate students through their academic program. With the appropriate documentation, the student will work with both

SAS and their graduate programs (normally their Graduate Chair and/or Course instructor) to ensure that appropriate academic accommodations to program requirements are arranged. These accommodations include individual counselling, alternative formatted literature, accessible campus transportation, learning strategy instruction, writing exams and assistive technology instruction.

See http://www.sdc.uwo.ca/ssd/

Please contact the course instructor if you require lecture or printed material in an alternate format or if any other arrangements can make this course more accessible to you. You may also wish to contact Student Accessibility Services (SAS) at (519) 661-2147 if you have any questions regarding accommodations.

Western University is committed to a thriving campus as we deliver our courses in the mixed model of both virtual and face-to-face formats. We encourage you to check out the Digital Student Experience website to manage your academics and well-being:
https://www.uwo.ca/se/digital/

Religious accommodation: Students should consult the University's list of recognized religious holidays, and should give reasonable notice in writing, prior to the holiday, to the Instructor and an Academic Counsellor if their course requirements will be affected by a religious observance. Additional information is given in the Western Multicultural Calendar: https://multiculturalcalendar.com/ecal/index.php?s=c-univwo

Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web sites:

`http://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_undergrad.pdf`

`https://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_grad.pdf`

Medical accommodation: if you are unable to meet a course requirement due to illness or other serious circumstances, you must ensure that you have valid medical or other supporting documentation. The Student Medical Certificate is available at
`https://www.uwo.ca/univsec/pdf/academic_policies/appeals/medicalform.pdf`

It is the student's responsibility to request accommodation and make alternative arrangements with the instructor.

Students who are in emotional/mental distress should refer to Mental Health@Western (`http://www.health.uwo.ca/mental_health`) for a complete list of options about how to obtain help.

Additional student-run support services are offered by the USC
http://westernusc.ca/services